(Almost) Certain Modus Ponens

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Introduction

I'll argue against:

Certain Modus Ponens: It's certain that φ. If φ, ψ. Therefore, it's certain that ψ.

Certainty Preservation: It's certain that φ. φ entails ψ. Therefore, it's certain that ψ.

Background

A while ago, I was thinking about counterexamples to

Chancy Modus Ponens: Probably φ. If φ, ψ. Therefore, probably ψ.

Certain Modus Ponens is a natural way to strengthen **Chancy Modus Ponens**. So I was wondering whether my counterexamples, inspired by McGee 1985, carry over. Here, I argue that they do, given plausible background assumption.

Chancy Modus Ponens

Horse Race. There's a horse race with three horses A, B and C. Horses A and C belong to team red, horse B to team blue. Horse A will win with 55% probability, horse B with 30% probability and horse C with 15% probability.

In this case, the following are true:

1. Probably a team-red horse wins.
2. If a team-red horse wins, then if it's not horse A who wins the race, it's horse C.

Chancy Modus Ponens

However, the following is false:

3. Probably, if it's not horse A who wins the race, it's horse C.

So Chancy Modus Ponens is invalid.

But what about Certain Modus Ponens?

Certainty

I will assume that **Certainty is Probability One**.

Worry: propositions with probability one may fail to be true. Classic example: throwing point-sized darts at a continuous dartboard.

However, it's natural to think of certainty as *maximal probability*, and that's one. Furthermore, decision theory tells you to bet *anything* on a probability one event, so it's reasonable to say that you are certain the event will occur.

Almost Certainty

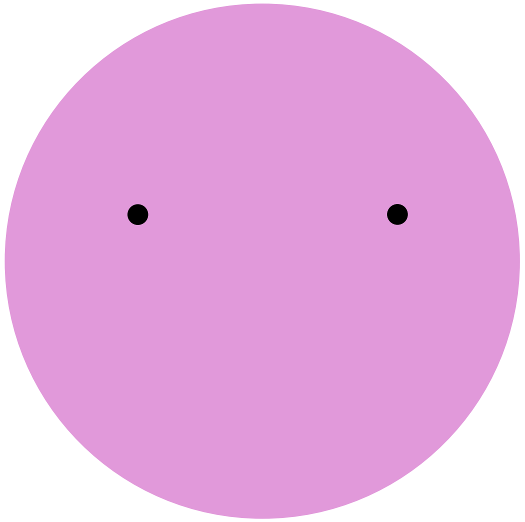
If you *really disagree*, that's ok. In this case, I invite you to read my argument as directed against:

Almost Certain Modus Ponens: φ has probability one. If φ, ψ. Therefore, ψ has probability one.

Almost Certainty Preservation: φ has probability one. φ entails ψ. Therefore, ψ has probability one.

The example

We are randomly throwing a point-sized dart at a continuous dartboard with two distinguished points **left** and **right**:



Certain Modus Ponens fails

We should accept:

1. It's certain that we won't hit left.
2. If we won't hit left, then if we hit either left or right, we hit right.

By **Certain Modus Ponens**, this entails

3. It's certain that if we hit either left or right, we hit right.

That doesn't sound right.

Certainty Preservation fails

We should accept:

1. It's certain that we won't hit left.

Plausibly, *we won't hit left* entails *if we hit either left or right, we hit right*. So by **Certainty Preservation**:

3. It's certain that if we hit either left or right, we hit right.

Again, that doesn't sound right.

Conditional Probability

But why does (3.) not sound right?

Plausibly, because we think that the conditional probability $P(\text{hit right} \mid \text{hit either right or left})$ is *not* one, but 1/2. Remember, we are throwing our dart randomly.

However, if we use the ratio definiton, $P(\text{hit right} \mid \text{hit either right or left})$ is undefined because the conditioning event has probability zero.

Primitive Conditional Probability

There are independent reasons to take conditional probabilities as primitive, which allows us to make sense of conditioning on probability zero events (Hájek 2003).

So we can maintain that there is a close connection between conditionals and conditional probabilities and say that $P(\text{hit right} \mid \text{hit either right or left}) = 1/2$.

Why Care?

It's *really strange* that **Certain Modus Ponens** and **Certainty Preservation** turn out to be false. They sound like they should be true!

Even if we deny that **Certainty is Probability One**, **Almost Certain Modus Ponens** and **Almost Certainty Preservation** sounds like they should be true.

However, there's more reasons to care.

McGee

McGee gives the following apparent counterexample to Modus Ponens:

> Having learned that gold and silver were both once mined in his region, Uncle Otto has dug a mine in his backyard. Unfortunately, it is virtually certain that he will find neither gold nor silver, and it is entirely certain that he will find nothing else of value. There is ample reason to believe
>
> If Uncle Otto doesn't find gold, then if he strikes it rich, it will be by finding silver.
>
> Uncle Otto won't find gold.
>
> Since, however, his chances of finding gold, though slim, are no slimmer than his chances of finding silver, there is no reason to suppose that
>
> If Uncle Otto strikes it rich, it will be by finding silver.

McGee

We can explain what's going on in McGee's case by saying that

1. (Almost) Certain Modus Ponens is invalid.
2. Modus Ponens is valid.
3. Certainty preservation fails.

We can construct a consequence relation that validates all of these.

Triviality

In a recent paper, Paolo Santorio (2021) suggests a "natural and seemingly harmless constraint" about the link between informational consequence and credence:

> “ informational consequence is certainty preserving. I.e., on any rational credence distribution, when the premises of an informational inferences have credence 1, the conclusion also has credence 1. ”

So Santorio endorses both **Certainty Preservation** and **Almost Certainty Preservation**.

Triviality

Santorio then uses **Certainty Preservation**, together with a few other assumptions, to prove that informational consequence collapses into classical consequence, which is contrary to its motivation.

However, as we have seen, we have reason to reject **Certainty Preservation**, thus avoiding triviality.

Broader lesson: it's very hard to come up with general principles linking credence and consequence, even credence one.

Closure

Plausibly, probability one is sufficient for belief. Perhaps it's also sufficient for knowledge. After all, we believe and know on the basis of probabilistic evidence, and probability one is the best we're ever going to get.

However, if **Certainty Preservation** fails, this means that belief/knowledge are *not* closed under logical consequence.

In the example: I believe/know that I won't hit left, but I don't believe/know that if I hit either left or right, I hit right.

References

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- Santorio, Paolo (2022). Trivializing Informational Consequence. *Philosophy and Phenomenological Research* 104 (2): 297-320.
- McGee, Vann (1985). A Counterexample to Modus Ponens. *Journal of Philosophy* 82 (9): 462-471.