Knowledge of Logical Truth

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Knowledge of Logical Truth

Topic: What makes a true belief in a logical truth *knowledge*?

Not: What makes a logical truth *true*?

This distinction gives a partial reply to one of Poincaré's objections to logicism about mathematics:

If every mathematical truth is equivalent to A = A, then all of mathematics is trivial!

Also not: What makes a logical truth *logical*?
And not: How does one *tell* whether a truth is a logical truth?
Nor: How does one tell whether a logical statement is true?

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Logical Truths a Problem for Accounts of Knowledge

- *Internalist* views (requiring access to reasons)
  - Sensible for cases of complex truths arrived at by explicit proof
  - Founder on knowledge of obvious logical truths like:

    "Either the robber escaped from the building or the robber did not escape from the building."

    WOS knows this without ability to give reasons.

    Even we would be hard-pressed to give a non-circular justification. (Carroll 1895)

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Logical Truths a Problem for Accounts of Knowledge

- *Externalist* views
  - **Safety** (neighborhood reliabilism): your true belief is knowledge if it couldn't easily be false. → Knowledge of logical truth requires of the subject only belief. TOO EASY
  - **Relevant Alternatives**: your true belief in p is knowledge if your evidence rules out the relevant -p possibilities. If p is a logical truth there are no -p possibilities. → Knowledge of logical truth requires of the subject only belief. TOO EASY

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Problem for Externalist Views, cont'd

- **Process Reliabilism**: Roughly, your true belief is knowledge iff it was formed by a reliable process.

1st problem (basic truths): Any process that simply makes the subject believe p when p is a logical truth will be a reliable process intuitively since it will always yield true beliefs. However, it is only a *conditionally* reliable process unless it is preceded by a process that determines for a given p whether it's a logical truth.

1. This seems to be more than what we do, even unconsciously (though I defer to cognitive scientists).

2. Plus, a general rule for this is not even possible beyond propositional logic. However, 1) that may also be right where we stop having easy knowledge of basic truths, and 2) we could be using special rules.

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Problem for Externalist Views, cont'd

- **Process Reliabilism**, cont'd:

2nd problem (less basic truths): Suppose one comes to believe a logical truth by deductive reasoning, a conditionally reliable process. Inferring a logical truth from a contingent truth counts, and ought to count, as a reliable process, since it will always yield a truth.

But **PR** says nothing that prevents the person who counts as knowing this way from withdrawing belief in the logical truth when the contingent truth goes false. Yet we would not count a person who would do that as knowing the logical truth.

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Problem for Externalist Views, cont'd

- **Tracking**: Roughly, your true belief in p is knowledge iff

- if p weren't true you wouldn't believe p [P(-b(p)/-p) high] and
- if p were true you would believe p [P(b(p)/p) high]

For p a necessary truth the first condition is trivially fulfilled (subjunctive conditional version) or undefined (conditional probability version).

The account reduces to the second condition. Not a disaster—the second condition is not trivial—but we can do better. Notice second condition roughly equivalent to believing "come what may".

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Responsiveness

- Knowledge is a kind of responsiveness to the truth.

- Responsiveness is the reason that tracking gives the best account of knowledge of contingent truth: contingent truths can change and one must be able to *follow* that. Tracking is following.

- A different kind of responsiveness is needed for a different kind of truth whose truth value never changes--necessary truth

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Overall Plan

- Define ***knowledge of logical implication*** as a certain kind of responsiveness to the relation between two statements when one implies the other.
- Define ***knowledge of (non-implicational) logical truth*** as responsiveness to the fact that the logical truth is *implied by every proposition*.
- The second step will involve a universal quantification of the condition of the first step. It signals the subject's appreciation of the place of logical truths in relation to other truths.

N.B.: This is modulo recursion clauses and other diddles.

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Knowledge of Logical Implication

S knows that $q_1, \ldots, q_n$ *imply* p iff

1) $q_1, \ldots, q_n$ imply p

2) S believes $q_1, \ldots, q_n$ imply p

3) $P((-b(q_1) \vee \ldots \vee -b(q_n))/-b(p)) > s$

4) $P(b(p)/b(q_1) . \ldots . b(q_n)) > s$, and

5) if 2) is fulfilled because of inference(s) S made from $q_1, \ldots, q_n$ to p, then every step of inference in the chain is one where S knows the premise implies the conclusion,

with s = 0.95 to fix ideas.

Note: 3), 4) are not tracking conditions.

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Knowledge of Logical Implication

E.g., let n = 1, so “q implies p” is at issue. A subject who believes that q implies p then knows it (roughly) only if:

3) if she didn’t believe p then she wouldn’t believe q, and
4) if she did believe q then she would believe p,

that is, only if she appreciates the special relation between these two propositions, shown in her dispositions to believe and not believe.

--This does not require her to have beliefs about or awareness of statements *about* that relation. Hence, the view is externalist.

--It makes it possible to count the MOS as having knowledge of obvious logical implications. All he needs to do is behave properly.

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Knowledge of Logical Implication

The requirement for knowing that q implies p is not a tracking relation.

--She is not required to track ‘q implies p’ (thus avoiding the problem of the original tracking view).

--Nor to track p, nor to track q; only her beliefs involved.

--We are not even requiring that her belief that p track her belief that q.

Why not?

--That q is true or p is true is not implied by ‘q implies p.’

--Logical implication is not a tracking relation either: ‘q implies p’ implies ‘if not p then not q,’ but does not imply ‘if not q then not p.’

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Knowledge of Logical Implication

q = There is a red barn in front of me.
p = There is a barn in front of me.
q implies p. Suppose I believe q implies p.

On this account, I know that q implies p iff

P(-b(q)/-b(p)) > 0.95
P(b(p)/b(q)) > 0.95

That is, in the probable circumstances in which I don't believe p I also don't believe q, and

in the probable circumstances in which I believe q I also believe p.

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Non-implicational Logical Truth

- To be properly responsive to logically true propositions is not to follow their truth values because their truth values never change.
- Rather, to know them one should be responsive to the special position they occupy among all propositions of the language.
- One property that describes this special position is that logical truths are *logically implied by every proposition*.
- It would be crazy to require for knowledge of a (non-implicational) logical truth that a person *know* that the logical truth is implied by every proposition. That requires a course in logic that the MOS hasn't taken.

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Non-implicational Logical Truth

- However, we don't have to require that a person *know* that the logical truth is implied by every proposition in order to require responsiveness to this fact.
- A person need not even believe that every proposition implies p in order to fulfill the following conditions:
  - For every set of propositions $\{q_1, \ldots, q_n\}$,
    $P((-b(q_1) \vee \ldots \vee -b(q_n))/-b(p)) > 0.95$
  - For all $q_1, \ldots, q_n$, none of which is -p,
    $P(b(p)/(b(q_1). \ldots .b(q_n))) > 0.95$

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Non-implicational Logical Truth

S knows p, a non-implicational logical truth, if S believes p and

(c′) For every set of propositions $\{q_1, \ldots, q_n\}$,
$P((-b(q_1) \vee \ldots \vee -b(q_n))/-b(p)) > 0.95$

(d′) For all $q_1, \ldots, q_n$, none of which is -p,
$P(b(p)/(b(q_1). \ldots .b(q_n))) > 0.95$

Note: If S knows p this way then she believes p *come what may* among her beliefs. This isn't come what may simpliciter, but that would require immortality.

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Non-implicational Logical Truth

However, we can't make (c′), (d′) necessary conditions for this kind of knowledge.

Imagine p a complex non-implicational logical truth one knows by deriving it from other logical truths. It is legitimate for one's belief in p to depend on one's belief that each step in the proof was legitimate. If one were to come to believe that step 3 was illegitimate then one would rightly withdraw belief in the conclusion of the proof, p. (d′) would wrongly penalize one for this disposition.

→ We need a recursion clause.

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Non-implicational Logical Truth

S knows p*, a non-implicational logical truth*, iff
S believes p and

either (base clause)

(c′) for every set of propositions $\{q_1, \ldots, q_n\}$,
$P((-b(q_1) \vee \ldots \vee -b(q_n))/-b(p)) > 0.95$ and

(d′) for all $q_1, \ldots, q_n$, none of which is -p,
$P(b(p)/(b(q_1). \ldots .b(q_n))) > 0.95$

or (recursion clause)

there are logical truths $q_1, \ldots, q_n$ none of which is p such that S knows that $q_1, \ldots, q_n$ logically imply p and S knows $q_1, \ldots, q_n$.

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Non-implicational Logical Truth

That is, S knows p, a non-implicational logical truth iff S believes p and

either

if S withdrew belief in p she wouldn't believe anything and

S has the belief-come-what-may property wrt p

or

there are logical truths, none of which is p, that S knows and that she knows logically imply p.

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No Knowledge of Logical Truth by Authority

- Non-implicational logical truth, p:
  - Base clause requires belief in p come what may. If you believe on basis of authority, then you would change your belief if you believed the authority changed his mind.
  - Recursion clause same as knowledge of logical implication
- Logical implication, q implies p:
  - If believed on authority, then you would fail to believe p when you believed q in the circumstance when you believe the authority has changed his mind. If the latter has > 0.05 probability, you fail condition (d).

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No Knowledge of Logical Truth by Authority

Apparent Exceptions:

- **Authority is like Odysseus** on the mast, who immunized you against future change in his orders.
  - There is a q, belief in which would make you withdraw belief in p: q = 'What Odysseus said before going up on the mast was -p'
  - That is, even this doesn't give you belief come what may.
- **Authority is linguistic community**

  If one would keep belief in p even if one believed the entire linguistic community had changed its mind, then one's belief is not *based* on the authority of the linguistic community.

  If one would lose belief when one believed the community changed its behavior, then one doesn't fulfill the condition for knowledge.

This doesn't mean you can't *learn* by authority.

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How general is the account?

Does the account work only for classical logic?

We exploited the fact that a logical truth is implied by every proposition. so the account will make sense in any logic in which this is true.

Put syntactically, the fact in question is that a logical truth is derivable from any statement of the language.

What is the independent purchase on what the logical truths of a system are? Say the logical truths are the theorems, those things derivable from the empty set, E.

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How general is the account?

The question is then whether those truths are derivable from any set of premises. All other possible premises imagined will be additions to E, so the question is whether adding any set of those could ruin the implication.

That is, we need: For all S and p, if E ⊢ p then E,S ⊢ p.

So, it looks like the account will be available in monotonic logic, where adding premises does not change the implication, but not in non-monotonic, where adding premises can ruin the implication. (Think of induction.)

However, what we need is only a special case of monotonicity with E as premise, so could you have this property for deductions with empty set as premise but not for other implications? Yes, it seems.

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How general is the account?

Suppose we have a monotonic logic in which this account of knowledge of logical truth is available. Why should we think this is the *best* account of knowledge of that kind of logical truth?

The logical truths do have this property, so in being responsive to this property you are being responsive to something about the logical truths.

Moreover, the logical truths are the only truths that will have this property (in classical and it seems in the other logics), so this requirement on knowledge is forcing you to respond to these special truths in a distinctive way.

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Other Necessary Truths

The view can be adapted for:

analytic truths depending on meaning –

substitute 'analytic truth' for 'logical truth' and fix language

mathematical truths – two options

1. all mathematical necessity is logical necessity
2. substitute 'mathematical truth' for 'logical truth' and fix language

laws of nature – two options

1. these are contingent → use the tracking conditions
2. substitute 'physically necessary truth' for 'logical truth' and fix laws of nature

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Summary

S knows q implies p only if (roughly)

if S were not to believe p she wouldn't believe q, and

if S were to believe q she would believe p.

S knows p, a non-implicational logical truth she believes, iff either

S believes it come what may, and

if S didn't believe it she wouldn't believe anything

or

S knows p follows from some logical truths that she knows.

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