Frequencies of Conditionals and Conditional Frequencies

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Abstract

There is an extensive list of analogies between an epistemic reading of indicative conditionals, seemingly talking about epistemic possibilities, and an *iterative* reading, seemingly talking about events or situations. For the epistemic reading, there is a long-standing problem stemming from the observation that probabilities of conditionals seem to be conditional probabilities. A parallel observation can be made for the iterative reading: frequencies of conditionals seem to be conditional frequencies. This leads to a hitherto unrecognized version of the problem. We present several triviality results involving frequencies of conditionals, and argue that a unified solution for probabilities and frequencies should be sought. We investigate which approaches facilitate such a unified solution, and argue that this desideratum favors views that do not identify probabilities of conditionals with probabilities of conditional propositions.

1 Introduction

Consider an indicative conditional like:

(1) If the die landed even, it landed on two or four.

This conditional has two readings. First, it can be used to talk about a specific die roll, whose outcome has not yet been revealed. By uttering the conditional, a speaker can convey information about what outcomes they regard as live epistemic possibilities. What they convey can plausibly be paraphrased as follows: in each epistemic possibility where the die landed even, it landed either two or four.[1] Let us refer to this as the *epistemic reading* of the conditional. Second, the conditional can be used to state a generalization about a

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[1]Many different theories interpret indicative conditionals along these lines; among them Kratzer (1981, 1986); Gillies (2004, 2009); Yalcin (2007); Bledin (2014); Starr (2014); Willer (2014, 2018); Moss (2015); Punčochář and Gauker (2020); Ciardelli (2021).

sequence of die rolls. By uttering the conditional a speaker conveys that on each occasion on which the die landed even, it landed on two or four. Let us refer to this as the *iterative reading* of the conditional.[2]

The two readings seem to work in a structurally analogous way. In both settings, the conditional is interpreted with respect to a set of cases: in the epistemic setting, these are epistemic possibilities—ways the world might be; in the iterative setting, they are occasions—something like actual events or situations. The *if*-clause effects a restriction of the set of cases, and the conditional as a whole claims that the consequent is true throughout the restricted set.

The analogy can be taken further. Both in the epistemic and in the iterative setting, we have operators that act as quantifiers over cases. For instance:

(2) a. The outcome must/might/can't have been odd.

b. The outcome was always/sometimes/never odd.

Both (2-a) and (2-b) can be analyzed as saying that the outcome was odd in all/some/none of the cases, where the cases are epistemic possibilities in (2-a) and occasions in (2-b).

Moreover, in both settings, conditionals can be used to restrict the domain of quantification of these operators:

(3) a. The outcome must have been odd if it was prime.

b. The outcome was always odd if it was prime.

Both (3-a) and (3-b) can be analyzed as saying that all cases where the outcome was prime are cases where it was odd.[3]

These analogies suggest that the semantics of modals and conditionals in the epistemic setting works very much analogously to the semantics of adverbs of quantification and conditionals in the iterative setting.

On the epistemic side, a very important role is played by the notion of subjective probability, or credence, which we will simply refer to as *probability*. Imagine that a die roll has just taken place but the outcome has not yet been revealed. Then we can ask what is the probability of (4).

(4) The outcome was odd.

The natural answer is 1/2. This is the ratio of favorable cases—cases where (4) is true—to the number of total cases, where cases are epistemic possibilities about the outcome.

[2] This reading is discussed and analyzed by Lewis (1975). An analysis of conditionals along these lines has also been attributed to Diodorus Cronus (Mates, 1949).

[3] For such analyses, see (Lewis, 1975; Kratzer, 1981, 1986).

On the iterative side, we find an analogue to probability in the notion of *frequency*. Imagine we are talking about a sequence of rolls: 3, 4, 3, 4, 2, 5. How frequently was (4) the case? Again the natural answer is 1/2 of the time. This is again the ratio of favorable cases—cases where (4) is true—to the number of total cases, where a *case* here is an occasion on which the die is rolled.

What these examples show is that probabilities and frequencies are computed similarly, except that for probabilities the set of cases is given by epistemic possibilities, and for frequencies cases are occasions.

The parallel extends to conditionals. Consider again the first scenario above. What is the probability of (5)?

(5) If the outcome was even, it was two or four.

The natural answer is 2/3. Here, this is the ratio of cases where the outcome was two or four to the number of cases where the outcome was even. This is nothing but the conditional probability of the consequent given the antecedent.

Now consider again the second scenario, where we are talking about a sequence of die rolls: 3, 4, 3, 6, 2, 5. What is the frequency of (5)—how frequently was it the case that if the outcome was even, it was two or four? The natural answer is again 2/3 of the time. This is again the ratio of the number of cases where the outcome was two or four to the number of cases where the number was even. This is nothing but the *conditional frequency* of the consequent given the antecedent. Again, probabilities and frequencies of conditionals seem to be computed in the same way, except that the relevant domains of quantification consist of entities with different status. In both cases, the probability/frequency of a conditional seems to be a conditional probability/frequency.

In the case of probabilities, the reported judgments about conditionals have been the source of a long-standing theoretical problem. The assumption that probabilities of conditionals generally align with the corresponding conditional probabilities, paired with standard assumptions about probabilities, leads to unacceptable conclusions. This is brought out by a host of *triviality results*, following up on Lewis' (1976) original result. Many different proposals have been made about how to account for the relevant intuitions while avoiding the paradoxical conclusions.

Much less attention, if any, has been paid to the problem of how to account for the relevant frequency judgments. Yet, as we will show, analogues of different brands of triviality results also arise with respect to frequencies.[4]

[4]The analogy between probability and frequency judgments about conditionals appear to extend to compound sentences, where a conditional is embedded under conjunction, disjunction or another conditional operator. These pose a well-known challenge in the literature on probabilities of conditionals. As an illustration of this analogy we can adapt an example from (Ciardelli and Ommundsen, 2022, §5) concerning probabilities of conjunctions of conditionals. Suppose we are talking about

We think that considering the problem of frequencies of conditionals can be illuminating in several ways. For one thing, it shows that the triviality problem is not inherently linked to the epistemic nature of probability. Furthermore, in the case of probabilities it is sometimes debatable what the relevant set of epistemic possibilities is. An advantage of considering frequencies is that, by contrast, it is often quite clear what the occasions being counted are. This rules out the possibility of relying on a tailor-made space of cases. Finally, considering frequencies gives us a criterion to assess existing solutions to the triviality problem: given that the facts about probabilities and frequencies are structurally parallel, it is natural to require theories about probabilities of conditionals to extend to frequencies. As we will see, however, several theories in fact fail to extend. We will argue that extending the scope of the problem to frequencies favors theories that deny that probabilities of conditionals are probabilities of (standard, bivalent) propositions.

The paper is structured as follows. §2 begins by presenting a natural set of assumptions about conditionals and their probabilities and frequencies. We then derive three triviality results from these assumptions. §3 considers the conclusions suggested by these results and proposes a diagnosis as to what goes wrong in the triviality proofs. §4 considers three responses to the probabilistic triviality results: Lewis-Jackson type pragmatic views, a contextualist view, and Edgington's expressivism. We argue that these views do not plausibly extend to the frequential case. By contrast, other approaches do extend to in a natural way, which is an argument in their favor. In §5 we show this for two theories: a three-valued approach to conditionals, and an account based on Attitude Semantics, related to expressivist approaches. §6 concludes.

2 Frequential Triviality

This section presents three triviality results for frequencies. We explain how they are related to probabilistic triviality results: our first two results are analogues of known probabilistic results, while the third is formally new and without a probabilistic analogue. First, we turn to the assumptions of these triviality results.

the following (striking) sequence of die rolls: 1, 2, 3, 4, 5, 6. How frequently was the following the case?

(i) If the outcome was even, it was a two, and if it was odd, it was either one or three.

The natural answer seems to be 1/2, as the sentence appears simply to say that the outcome was one, two, or three. But given that the frequency of the first conjunct in (i) is intuitively 1/3, frequencies of compounds of conditionals thus would appear to violate laws of standard probability theory. These puzzling frequency judgments are completely analogous to the probability judgments discussed by Ciardelli and Ommundsen.

While we think such compounds are important, we will here limit our discussion to the case of simple, unembedded conditionals. As we shall argue, there are important lessons about the triviality problem to draw from frequencies of conditionals which arise already in this simpler and less contentious setting.

2.1 Background assumptions.

We start by spelling out a standard set of assumptions about how probabilities attach to sentences in context. In combination with the claim that probabilities of conditionals are conditional probabilities, these lead to triviality results.

First, a declarative sentence A in a context expresses a proposition $\mathbf{A}^c$, which is true or false relative to a possible world. This proposition therefore determines a set of possible worlds—those in which it is true—and can for our purposes be identified with this set (although nothing we say hinges on this identification). At least in the finite setting, it is natural to model an epistemic state as a pair $s = \langle L, m \rangle$ where L is a set of worlds—the live epistemic possibilities—and m assigns to each world $w \in L$ a probability mass $m(w)$—the probability that w is the actual world. The probability of a proposition $\mathbf{A}$, $p_s(\mathbf{A})$, is then the probability that $\mathbf{A}$ is true, i.e., that the actual world is one of those in $\mathbf{A}$. The probability of $\mathbf{A}$ thus equals the sum of the probability masses of live possibilities $w \in L$ such that $w \in \mathbf{A}$:

$$p_s(\mathbf{A}) = \sum_{w \in L \cap \mathbf{A}} m(w)$$

From this it follows easily that the Kolmogorov axioms hold for the algebra of propositions. The probability of a sentence A in a context c is simply the probability that the proposition expressed, $\mathbf{A}^c$, is true:

$$P_s^c(\mathsf{A}) = p_s(\mathbf{A}^c)$$

In the following, we will take the context to be fixed and suppress reference to it, except where we explicitly discuss accounts appealing to context dependence.

We also have conditional probabilities. These arise from considering probabilities relative to a restricted domain of possible worlds. Given a state $s = \langle L, m \rangle$ and a proposition $\mathbf{A}$ with $p_s(\mathbf{A}) > 0$, the restriction of s to $\mathbf{A}$ results in a new epistemic state $(s + \mathbf{A}) = \langle L \cap \mathbf{A}, m_{\mathbf{A}} \rangle$ where $m_{\mathbf{A}}$ is the same as m, modulo rescaling by a factor $1/p_s(\mathbf{A})$ to ensure the probability of the entire space remains equal to 1. More explicitly, for each $w \in L \cap \mathbf{A}$, $m_{\mathbf{A}}(w) = m(w)/p_s(\mathbf{A})$. The conditional probability of $\mathbf{B}$ given $\mathbf{A}$ is defined as the probability of $\mathbf{B}$ after restriction with $\mathbf{A}$, $p_{s+\mathbf{A}}(\mathbf{B})$. Using the definitions, it is easy to show that conditional probabilities of propositions can always be computed by the ratio formula:

$$p_{s+\mathbf{A}}(\mathbf{B}) = \frac{p_s(\mathbf{A} \cap \mathbf{B})}{p_s(\mathbf{A})}$$

Conditional probabilities of sentences in context are defined similarly as probabilities under restriction:

$P_s(\mathsf{B}|\mathsf{A}) = P_{s+\mathbf{A}}(\mathsf{B})$.

To these assumptions about how probabilities attach to sentences in context, we add what we call Adams' Thesis: the assumption, central to the work of Adams, that probabilities of conditionals are conditional probabilities.[5]

Adams' Thesis: For every epistemic state s and all sentences A, B with $P_s(\mathsf{A}) > 0$,

$$P_s(\mathsf{A} \Rightarrow \mathsf{B}) = P_s(\mathsf{B}|\mathsf{A})$$

where $\mathsf{A} \Rightarrow \mathsf{B}$ denotes the indicative conditional with antecedent A and consequent B.[6,7]

As we will review in this section, from these assumptions about probabilities, blatantly false conclusions can be deduced. Before turning to that, however, let us consider the counterparts of these assumptions in the setting of frequencies.

First, many sentences are naturally assessed as being true or false, not just relative to a world, but also relative to a particular occasion. For instance, in one and the same world, sentence (4) can be true relative to some die rolls, and false relative to others. In this way, such a sentence can be associated (relative to a world, that we can suppose fixed) with a set of occasions—those at which it is true. We will call this set of occasions the *occasion type* of A and denote it $\mathcal{A}$. Formally, the occasion type associated with A is the counterpart in the frequential setting of the proposition expressed by A.

In analogy with probability, we will assign frequencies in the first place to occasion types, and then derivatively to sentences. In a finite domain k of occasions, the frequency of an occasion type $\mathcal{A}$ is the proportion of occasions in k where $\mathcal{A}$ is true:

$$f_k(\mathcal{A}) = \frac{\#(k \cap \mathcal{A})}{\#k}$$

It is easy to show that, based on this definition, the Kolmogorov axioms hold in the space of occasion types (i.e., sets of occasions). The frequency of the sentence A in k is just the frequency of the associated occasion

[5] See (Adams, 1975). This thesis (and sometimes close variants) is also often called Stalnaker's Thesis, after Stalnaker (1970) who briefly held the view.

[6] We take this to govern the epistemic reading of *bare* indicative conditionals—i.e., indicative conditionals whose consequent does not contain a modal or another item that the *if*-clause could serve to restrict. Conditionals involving such operators present a systematic ambiguity depending on whether or not the *if*-clause restricts the modal, see (Geurts, 2004). Adams' Thesis applies only to one of their readings. We take such an ambiguity to arise also for conditionals involving the modal *will* (see [redacted for peer review]), which is why we stick to conditionals in the past tense throughout the paper.

[7] Some see Adams' Thesis in the first place as a rationality constraint. However, that Adams' Thesis is a rationality constraint is implausible for its frequential version: unlike assignments of credence, assignments of frequency are not naturally seen as more or less rational, except insofar as they may or may not agree with the facts.

type $\mathcal{A}$:

$$F_k(\mathsf{A}) = f_k(\mathcal{A})$$

Conditional probabilities have a natural counterpart in conditional frequencies. These arise as frequencies relative to a restricted domain of occasions. Given an occasion domain k and an occasion type $\mathcal{A}$, we can consider the restricted domain $k + \mathcal{A} := k \cap \mathcal{A}$. Provided this is non-empty, we can define the conditional frequency of $\mathcal{B}$ given $\mathcal{A}$ as a frequency in this restricted domain, $f_{k+\mathcal{A}}(\mathcal{B})$. It is easy to show that conditional frequencies, like conditional probabilities, can be computed by a ratio formula:

$$f_{k+\mathcal{A}}(\mathcal{B}) = \frac{f_k(\mathcal{A} \cap \mathcal{B})}{f_k(\mathcal{A})}$$

Conditional frequencies for sentences are defined similarly as frequencies under restriction: $F_k(\mathsf{B}|\mathsf{A}) = F_{k+\mathcal{A}}(\mathsf{B})$.

The analogue of Adams' Thesis for frequencies is the following claim, identifying frequencies of conditionals with conditional frequencies.

Frequential Adams' Thesis: For any occasion domain k and sentences A, B with $F_k(\mathsf{A}) > 0$,

$$F_k(\mathsf{A} \Rightarrow \mathsf{B}) = F_k(\mathsf{B}|\mathsf{A})$$

As in the case of probabilities, from these assumptions about frequencies some patent falsehoods can be derived, as we are going to see.

2.2 Triviality via cardinality

Probabilistic case. The following is based on an example due to Hájek and his "wallflower" result.[8] Imagine a case of uncertainty between three exhaustive and mutually exclusive outcomes, x, y and z, which are taken to be equally likely. It is natural to model this situation by means of a state where three possibles worlds, w_x, w_y, and w_z, are all assigned equal probability mass, 1/3. What is the probability of (6)?

(6) If the outcome is not x, it is y.

The natural answer is the corresponding conditional probability, 1/2. We have assumed that the probability of (6) is the probability of the proposition it expresses. This yields a contradiction: all propositions in our model have probability a multiple of 1/3, so no proposition has probability 1/2.

[8] For the example see (Hájek, 2012); for the general wallflower result see (Hájek, 1989).

This argument relies crucially on the model's specification of the epistemic possibilities. One might resist the argument by finding fault with this specification. Perhaps the model we gave is too coarse-grained to represent the scenario we described. In any case of uncertainty about three options, were we serious about describing the possibilities involved we might grant that really each world in the model above corresponds to an equivalence class of worlds, the countless worlds which agree on the outcome but disagree on other issues. If so, a conditional could express a proposition which cuts across these classes. This proposition is left out of the coarse model above, which explains our problem of finding a suitable proposition for our conditional. As we will see, however, an analogous response is not available in the frequential setting.

Frequential case. Imagine again that we are talking about a sequence of die rolls:

$$3, 4, 3, 4, 2, 5$$

How frequently did the following happen?

(7) If the outcome was prime, it was odd.

The natural answer is 3/4 of the times, matching the corresponding conditional frequency. The relevant domain of occasions is, by all appearances, the six die rolls. We have assumed that the frequency of (7) is the frequency of some occasion type from this domain. This is impossible: all occasion types have frequency a multiple of 1/6, so no occasion type has frequency 3/4.[9]

This problem is essentially the same as the one we discussed in the probabilistic setting. But the line of response that we sketched above does not seem viable here. We may wonder whether a situation of uncertainty about three outcomes is well-modeled by an epistemic state over three worlds; but there seems little doubt about what the relevant domain of occasions is in our setting: it is the set of the six die rolls. What else could it be?

2.3 Triviality via conditionalization

Probabilistic case. Let us recall a version of Lewis' (1976) first triviality result. Consider an epistemic state s and sentences A and B such that, writing P for P_s, we have $P(\mathsf{A} \wedge \mathsf{B}), P(\mathsf{A} \wedge \neg\mathsf{B}) > 0$.[10] First, by the law of total probability (which follows from the Kolmogorov axioms and the ratio formula for conditional

[9] This problem is pervasive: it arises whenever there are sentences A and B such that the conditional frequency of A on B is not a multiple of $1/n$, where n is the total number of occasions. If the domain contains at least three occasions and any occasion type is expressed by some sentence, sentences A and B with the (un)desired properties can always be found.

[10] For simplicity throughout this section we presuppose that Boolean connectives have the classical truth-functional effects. But we could dispense with this assumption and work instead at the level of propositions or occasion types.

probability) and the fact that $P(\mathsf{B}), P(\neg\mathsf{B}) > 0$, we have:

$$P(\mathsf{A} \Rightarrow \mathsf{B}) = P(\mathsf{A} \Rightarrow \mathsf{B}|\mathsf{B}) \cdot P(\mathsf{B}) + P(\mathsf{A} \Rightarrow \mathsf{B}|\neg\mathsf{B}) \cdot P(\neg\mathsf{B})$$

By Adams' Thesis for $P(\cdot|\mathsf{B})$ and $P(\cdot|\neg\mathsf{B})$ and the fact that $P(\mathsf{A} \wedge \mathsf{B}), P(\mathsf{A} \wedge \neg\mathsf{B}) > 0$, this equals

$$P(\mathsf{B}|\mathsf{A} \wedge \mathsf{B}) \cdot P(\mathsf{B}) + P(\mathsf{B}|\mathsf{A} \wedge \neg\mathsf{B}) \cdot P(\neg\mathsf{B})$$

which boils down to

$$1 \cdot P(\mathsf{B}) + 0 \cdot P(\neg\mathsf{B}) = P(\mathsf{B})$$

We have established that in any epistemic state where A is probabilistically compatible with both B and $\neg\mathsf{B}$, $P(\mathsf{A} \Rightarrow \mathsf{B}) = P(\mathsf{B})$. But this is patently false: for instance, in a die roll scenario, the probability of (8) is 1/3, which is different from the probability of the consequent (1/6).

(8) If the outcome is even, it is two.

Frequential case. We saw that under the above assumptions, in a finite domain of occasions, frequencies satisfy the Kolmogorov axioms, and conditional frequencies are given by a ratio formula. This allows us to give a Lewisian triviality proof for frequencies.[11] To see how the argument runs, suppose we are in a non-trivial case: we have a finite occasion set k and sentences A and B such that A is true on some B-occasion and on some $\neg\mathsf{B}$-occasion. Then first by the law of total probability we have:

$$F_k(\mathsf{A} \Rightarrow \mathsf{B}) = F_k(\mathsf{A} \Rightarrow \mathsf{B}|\mathsf{B}) \cdot F_k(\mathsf{B}) + F_k(\mathsf{A} \Rightarrow \mathsf{B}|\neg\mathsf{B}) \cdot F_k(\neg\mathsf{B})$$

By Adams' Thesis for frequencies and the assumption that the number of $\mathsf{A} \wedge \mathsf{B}$ occasions and the number of $\mathsf{A} \wedge \neg\mathsf{B}$ occasions are both positive, the right hand side can be written:

$$F_k(\mathsf{B}|\mathsf{A} \wedge \mathsf{B}) \cdot F_k(\mathsf{B}) + F_k(\mathsf{B}|\mathsf{A} \wedge \neg\mathsf{B}) \cdot F_k(\neg\mathsf{B})$$

which again reduces to

$$1 \cdot F_k(\mathsf{B}) + 0 \cdot F_k(\neg\mathsf{B}) = F_k(\mathsf{B})$$

We have established that whenever A and B are such that there are $\mathsf{A} \wedge \mathsf{B}$ occasions and well as $\mathsf{A} \wedge \neg\mathsf{B}$ occasions, the frequency of the conditional $\mathsf{A} \Rightarrow \mathsf{B}$ is just the frequency of B. But this is a patent falsehood.

[11] In a similar way, we could give analogues of the triviality results by Bradley (2000), Milne (2003), and Fitelson (2015).

For a counterexample, suppose we made the sequence of die rolls 3, 4, 1, 2, 5, 6. Then the frequency of (9) is 1/3, which is different from the frequency of the consequent (1/6).

(9) If the outcome is even, it is two.

2.4 Triviality via multiplicities

Given a sentence which is true or false relative to occasions, it is natural to ask, not only how frequently it was the case, but also *how many times* it was the case. We will refer to this number as the *multiplicity* of the sentence in a given domain of occasions. For example, imagine again we are talking about the sequence of die rolls

$$3, 4, 3, 4, 2, 5$$

How many times was (10) the case?

(10) The outcome was prime.

The natural answer is 4 times. We thus say that the multiplicity of (10) in this scenario is 4.

There is an obvious extension of our assumptions about frequency to the case of multiplicities. The multiplicity of a sentence in domain k is just the number of occasions in k on which it is true. For any sentence A and finite occasion set k,

$$M_k(\mathsf{A}) := \#(k \cap \mathcal{A})$$

We can now deduce the following natural bridge principle relating frequencies and multiplicities: the frequency of a sentence is just its multiplicity divided by the total number of occasions.

F-M Bridge. For any sentence A and occasion set k,

$$F_k(\mathsf{A}) = \frac{M_k(\mathsf{A})}{\#k}$$

Conditionals too are naturally assessed for multiplicities, and intuitions about the correct answers are systematic. Given the sequence of outcomes above, how many times was (11) the case?

(11) If the outcome was even, it was prime.

The intuitively correct answer is: one time. This is the number of occasions on which the outcome was both even and prime. This is nothing but the *conditional multiplicity* of the consequent on the antecedent, i.e., the multiplicity of the consequent in the domain obtained by restricting to those occasions where the antecedent is true.

Multiplicities thus seem to satisfy a version of Adams' Thesis. Let us denote the conditional multiplicity of B given A by the notation $M_k(\mathsf{B}|\mathsf{A})$, defined as:

$$M_k(\mathsf{B}|\mathsf{A}) = M_{k+\mathcal{A}}(\mathcal{B})$$

Then the analogue of Adams' Thesis for multiplicities states that for every domain of occasions k and sentences A, B:

$$M_k(\mathsf{A} \Rightarrow \mathsf{B}) = M_k(\mathsf{B}|\mathsf{A})$$

Exploiting the tight relation between multiplicities and frequencies, we can derive a strong triviality result which does not have a counterpart in the probabilistic setting. Suppose we have sentences A, B and a finite domain of occasions k such that A is false on some occasion in k and A, B are true together on some other occasion in k. By our assumption about frequencies of conditionals, the frequency of $\mathsf{A} \Rightarrow \mathsf{B}$ in k is the conditional frequency of B given A:

$$F_k(\mathsf{A} \Rightarrow \mathsf{B}) = F_k(\mathsf{B}|\mathsf{A}) = \frac{\#(k \cap \mathcal{A} \cap \mathcal{B})}{\#(k \cap \mathcal{A})}$$

By our assumption about multiplicities of conditionals, the multiplicity of $\mathsf{A} \Rightarrow \mathsf{B}$ is the number of B-occasions among the A-occasions in k:

$$M_k(\mathsf{A} \Rightarrow \mathsf{B}) = M_k(\mathsf{B}|\mathsf{A}) = M_{k+\mathcal{A}}(\mathcal{B}) = \#(k \cap \mathcal{A} \cap \mathcal{B})$$

This is inconsistent with the F-M bridge principle. By the bridge principle and the two conclusions above,

$$\frac{\#(k \cap \mathcal{A} \cap \mathcal{B})}{\#(k \cap \mathcal{A})} = F_k(\mathsf{A} \Rightarrow \mathsf{B}) = \frac{M_k(\mathsf{A} \Rightarrow \mathsf{B})}{\#k} = \frac{\#(k \cap \mathcal{A} \cap \mathcal{B})}{\#k}$$

Given that $\#(k \cap \mathcal{A} \cap \mathcal{B}) \neq 0$ by assumption, it follows that

$$\#(k \cap \mathcal{A}) = \#k$$

But since k is finite, this implies $k \cap \mathcal{A} = k$, which means that A is true on all occasions in k, contrary to assumption.

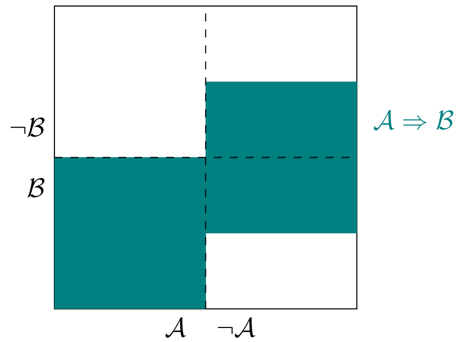

Figure 1: A hypothetical occasion type of $\mathsf{A} \Rightarrow \mathsf{B}$.

This triviality result can be visually explained as follows, by help of Figure 1.[12] For a given a conditional $\mathsf{A} \Rightarrow \mathsf{B}$, which occasions lie in its occasion type, $\mathcal{A} \Rightarrow \mathcal{B}$? A partial answer (related to the strong centering principle) might seem plausible: within the $\mathcal{A}$-region, the conditional is true on all $\mathcal{B}$-occasions and false on all $\neg\mathcal{B}$-occasions. Let us suppose this for concreteness, though the assumption is not needed for the result. We now see that our two Adams' Theses put incompatible constraints (up to triviality) on the occasion type $\mathcal{A} \Rightarrow \mathcal{B}$. First, it follows that since the frequency of $\mathsf{A} \Rightarrow \mathsf{B}$ is the conditional frequency $F_k(\mathsf{A} \wedge \mathsf{B})/F_k(\mathsf{A})$, the conditional must be true on an appropriately large number of occasions in the $\neg\mathcal{A}$ region, in addition to throughout the $\mathcal{A}\wedge\mathcal{B}$-region. Otherwise, the frequency of the conditional would be too low to satisfy Adams' Thesis for frequencies. Second, however, according to Adams' Thesis for multiplicities, the total number of occasions on which the conditional is true is precisely the number of occasions in $\mathcal{A}\wedge\mathcal{B}$. Thus the conditional cannot be true throughout $\mathcal{A} \wedge \mathcal{B}$ and, in addition, somewhere in the $\neg\mathcal{A}$-region; then it's multiplicity would be higher than what Adams' Thesis for multiplicities requires. Given our background assumptions, Adams' Thesis for frequencies gives us a relative measure of the size of $\mathcal{A} \Rightarrow \mathcal{B}$, Adams' Thesis for multiplicities gives us an absolute measure of the size of $\mathcal{A} \Rightarrow \mathcal{B}$. This results shows us that in non-trivial cases, no occasion type over k can simultaneously take both measures.

Notice that this result does not involve conditionalization in the way Lewis-type results do. The assumption that frequencies and multiplicities of conditionals are conditional frequencies and multiplicities was used only relative to the unrestricted domain k. As such, this triviality result is formally of the same kind as what are called *synchronic* triviality results in the probabilistic setting. As we argue below, this result therefore poses a challenge to views which take the hidden fallacy in most triviality arguments to involve conditionalization.

This concludes our survey of how triviality results arise in the frequential setting. We now consider what the response to these results should be.

[12]The visualization is inspired by Khoo and Santorio (2018).

3 Lessons of the frequential triviality results

What is the lesson of the triviality results? We think that looking at the frequential setting can help answering this question.

Consider again our first triviality result in §2.2. Given that, as we discussed above, the modeling of the scenario seems beyond reasonable doubt, the contradiction arises from simultaneously requiring that frequencies of conditionals amount to conditional frequencies, and that they amount to the frequency of some occasion type (since there are many conditional frequencies which are not the frequency of any occasion type). Intuitions support the identification of frequencies of conditionals with conditional frequencies, and the relevant intuitions are robust upon reflection. This leads to the conclusion that these frequencies are not the frequencies of occasion types. This seems intuitively plausible: when we answer questions about frequencies of conditionals, we just seem to compute frequencies in a restricted domain of occasions rather than assessing the conditional itself as true or false relative to each occasion. Given the structural analogy between the frequential and the probabilistic setting, this suggests an analogous diagnosis about probabilities. To compute the probability of a conditional is not to compute the probability of a proposition, but to compute a probability relative to a restricted set of epistemic possibilities.

Consider now the third triviality result in §2.4. The problematic chain of identities was:

$$\frac{\#(k \cap \mathcal{A} \cap \mathcal{B})}{\#(k \cap \mathcal{A})} = F_k(\mathsf{A} \Rightarrow \mathsf{B}) = \frac{M_k(\mathsf{A} \Rightarrow \mathsf{B})}{\#k} = \frac{\#(k \cap \mathcal{A} \cap \mathcal{B})}{\#k}$$

Of these identities, the first and the last are given by versions of Adams Thesis that seem strongly supported by intuition. By contrast, the second identity—an instance of the F-M bridge principle—clashes with intuition. For example, consider the sequence of rolls

$$3, 4, 3, 4, 2, 5$$

and the conditional:

(12) If the outcome was even, it was two.

The frequency of (12) is $1/3$, its multiplicity is 1, the total number of occasions is 6, and we have $1/3 \neq 1/6$, in contradiction to the second identity.

But as this identity is an instance of the F-M bridge principle, it can be deduced from the simple assumption that the multiplicity of a sentence is the number of occasions on which it is true, and its frequency the ratio of this number to the total number of occasions. These, then, are the assumptions that should be rejected in the case of conditionals. Once again, our results point to the conclusion that there is no occasion type $\mathcal{C}$ such that judging the multiplicity and the frequency of a conditional $\mathsf{A} \Rightarrow \mathsf{B}$ amounts to judging how many times and how often $\mathcal{C}$ was the case.

This conclusion also gives us a plausible diagnosis for what goes wrong in the second triviality result in §2.3. We saw that our construal of frequencies of occasion types implies that such frequencies obey the Kolmogorov axioms, and our construal of conditional frequencies of occasion types implies that these are given by the ratio formula. In combination with the assumption that the (conditional) frequency assigned to a conditional sentence $\mathsf{A} \Rightarrow \mathsf{B}$ is the (conditional) frequency of a corresponding occasion type, this implies the validity of the following instance of the law of total probability:

$$F_k(\mathsf{A} \Rightarrow \mathsf{B}) = F_k(\mathsf{A} \Rightarrow \mathsf{B}|\mathsf{B}) \cdot F_k(\mathsf{B}) + F_k(\mathsf{A} \Rightarrow \mathsf{B}|\neg\mathsf{B}) \cdot F_k(\neg\mathsf{B})$$

That this is the problematic identity can be confirmed by considering intuitions about a specific case. Consider again the conditional (12) and the sequence of die rolls 3, 4, 3, 4, 2, 5. The frequency of the conditional is 1/3. However, in restriction to the set of cases where the outcome is 2, the frequency of the conditional is 1. And in restriction to the set of cases where the outcome is not 2, its frequency is 0. Thus, the above equation yields the false conclusion

$$1/3 = 1 \cdot 1/6 + 0 \cdot 5/6 = 1/6$$

However, if frequencies of conditionals are not frequencies of some corresponding occasion types, this conclusion can be avoided: for then such frequencies do not necessarily validate the Kolmogorov axioms and the ratio formula, and thus do not necessarily obey the law of total probability.[13]

Extending the scope of the problem from probabilities to frequencies thus brings out more clearly where the source of the triviality problem lies, namely, in assuming that probabilities and frequencies attach to conditionals in the same way they attach to ordinary descriptive sentences, i.e., through a corresponding proposition or occasion type expressed by the conditional.

Other diagnoses as to the source of the triviality problem have of course been proposed. In the next section, we argue that some well-known proposals do not extend to the iterative side of the problem.

[13] Note that this amounts to rejecting a direct application of probability theory to conditionals, but not to a blanket rejection of standard probability theory, which is retained as a theory of probabilities of propositions and frequencies of occasion types. In fact, standard probability theory in a sense includes a theory of the probabilities and frequencies of conditionals, once these are identified with conditional probabilities and frequencies.

4 Proposals that do not extend

Frequencies and probabilities of conditionals are, formally speaking, almost completely analogous. The same goes for several of their triviality results. With these results we are dealing with two manifestation of one and the same problem, or so it seems to us. A solution to this problem—i.e., an account of why we have the relevant judgments and where the triviality proofs go wrong—should therefore apply to frequencies and probabilities in a unified way.

In this section we discuss how three proposed solutions to the triviality problem fail to carry over to the frequential setting.

4.1 Lewis-Jackson pragmatic theories

According to Lewis (1976) and Jackson (1987), indicative conditionals have the usual material truth conditions. As a consequence, their probabilities rarely match the corresponding conditional probabilities. Our intuitions are to be explained as the result of a confusion between probability and another quantity, *assertability* (or, for Jackson, assert*i*bility), which quantifies the degree to which an assertion of the sentence is justified. This confusion is motivated by the fact that, for many sentences, assertability goes by probability (or so the story goes). However, conditionals are special: for them, assertability departs from probability and instead goes by the conditional probability of the consequent given the antecedent (the two theories come with different stories about why that would be).

How would this story work for the case of frequencies? Presumably, the account would have to postulate that our intuitions about frequencies are also motivated by a confusion between frequency and assertability. But this is really implausible. Subjective probability quantifies one's confidence that the sentence is true, and as such, it may well bear on assertability. But frequency is a simple extensional notion, quantifying how often something was the case within a sequence of occasions. How could it be related to assertability? If we roll a die and obtain the sequence 3, 1, 1, 5, 2, then the outcome was odd with a frequency of 80%. Does this make the sentence "the outcome was odd" highly assertible with respect to the sequence? No! In fact an utterance of this sentence seems hardly interpretable if it is not referred to a particular rolling occasion; it can perhaps be interpreted as a generalization throughout all rolls, but as such it is clearly not highly assertible. Thus, this kind of response to the triviality results does not seem viable in the frequential case.

4.2 Contextualism

According to contextualist accounts, triviality results are to be avoided by appreciating the context-sensitivity of conditionals.[14] On these accounts, conditionals express propositions and the probability of a conditional is the probability of the proposition it expresses. However, what proposition a conditional expresses depends in a crucial way on the context in which it is used; in particular, it depends on some epistemic part of the context like a salient information state or piece of evidence. It is therefore crucial to take into account context explicitly when theorizing about probabilities of conditionals.

Recall that we denote by $P^c_s(\mathsf{A}) = p_s(\mathbf{\mathsf{A}}^c)$ the probability of the proposition expressed by A in context c, assessed on the basis of the information state s. Contextualist views assume that a context c determines a relevant information state s_c. How a context determines a corresponding information state is an important and potentially tricky issue, but we may set aside this worry here.[15] Then, the contextualist version of Adams' Thesis says that the probability of a conditional in a context, assessed with respect to the information state relevant in that same context, is the corresponding conditional probability.

Contextual Adams' Thesis: In any context c with $P^c_{s_c}(\mathsf{A}) > 0$,

$$P^c_{s_c}(\mathsf{A} \Rightarrow \mathsf{B}) = P^c_{s_c}(\mathsf{B}|\mathsf{A}).$$

The contextualist argues that this thesis is at once strong enough to explain the data motivating Adams' Thesis, and weak enough to avoid the triviality results. Important for avoiding triviality is that on this view, Adams' Thesis does not generally hold under conditionalization: we have $P^c_{s_c}(\mathsf{B} \Rightarrow \mathsf{C}|\mathsf{A}) = P^c_{s_c+\mathbf{\mathsf{A}}}(\mathsf{B} \Rightarrow \mathsf{C})$, but since the information state $s_c + \mathbf{\mathsf{A}}$ resulting from conditionalization is not the state associated with the context c, we cannot apply Contextual Adams' Thesis to conclude that this is equal to $P^c_{s_c+\mathbf{\mathsf{A}}}(\mathsf{C}|\mathsf{B})$, that is, to $P^c_{s_c}(\mathsf{C}|\mathsf{A} \wedge \mathsf{B})$.

Now let us see how this view can be applied to the iterative setting. The idea of contextualism carries over straightforwardly: we can assume that it is only relative to a context c that a sentence A determines a corresponding occasion type $\mathcal{A}^c$, which is perhaps dependent on the contextually relevant set of occasions k_c. Let us pause to note that, already at this point, the view is less plausible than it is in the epistemic realm. It requires that, relative to a context, a sentence is true or false on each particular occasion. But now take

(13) If the outcome is even, it is prime.

[14] An early paper influential for this tradition is van Fraassen's (1976). For examples of later work in this tradition, see (Bacon, 2015; Douven and Verbrugge, 2013; Mandelkern and Khoo, 2019; Boylan and Schultheis, 2022).

[15] In fact, we have already simplified slightly. Some approaches, like Bacon's, associate with contexts a set of probability functions, not a single function. This issue does not bear on our points below.

and suppose the context determines the sequence of outcomes 2, 6, 5, 3, 6. Relative to the particular occasion in which the outcome was 5, is (13) true or false? It is hard to see what about the context could determine an answer.

Setting aside this worry, denote by $F^c_k(\mathsf{A}) = f_k(\mathcal{A}^c)$ the frequency of the occasion type determined by A in context c, assessed within the domain of occasions k. Analogously to the probabilistic case, we may assume that a context determines a privileged domain of occasions, k_c. A contextual Adams' Thesis for frequencies now claims that the frequency of a conditional in the specific domain of occasions k_c determined by the context equals the corresponding conditional frequency.

Contextual Adams' Thesis for Frequencies: In any context c where $F^c_{k_c}(\mathsf{A}) > 0$,

$$F^c_{k_c}(\mathsf{A} \Rightarrow \mathsf{B}) = F^c_{k_c}(\mathsf{B}|\mathsf{A})$$

This restricted version of the thesis avoids the frequential analogue of Lewis' proof (our second triviality result) by denying that Adams' Thesis is applicable under conditionalization. However, the restricted version does not help with the other two triviality results, which can be run in a fixed context and do not involve conditionalization, and which therefore still arise for the contextualist.

For the first triviality result, suppose in a context c we are talking about a sequence of die rolls, 3, 4, 6, 2, 5. Then the contextually given domain of occasions is presumably the set of these rolls (what else?), and then the contextual thesis implies that the frequency of (14) is 2/3.

(14) If the outcome was even, it was two or four.

This is intuitively the right frequency, but this clashes with the assumption that this is the frequency of an occasion type, since the frequency of any occasion type in the given domain is a multiple of 1/5.

Contextualism also does not avoid the third triviality result from multiplicities. First, faced with the observations on multiplicities, the view should presumably be extended in the obvious way from frequencies to multiplicities. It should assume that multiplicities of sentences are multiplicities of occasion types, and that multiplicities satisfy a contextualist Adams' Thesis:

$$M^c_{k_c}(\mathsf{A} \Rightarrow \mathsf{B}) = M^c_{k_c}(\mathsf{B}|\mathsf{A})$$

These assumptions made, the crucial observation is then that the third triviality argument takes place entirely within a single context. We can therefore reproduce exactly the reasoning in §2.4 with frequencies and multiplicities computed relative to the context c and the associated occasion domain k_c. We thereby reach the paradoxical conclusion that there cannot be two sentences A, B such that relative to c, A is false on some occasion in k_c and A and B are true together on some other occasion in k_c.

Thus, an attempt to extend the contextualist solution to frequencies highlights two difficulties. First, the assumptions of the approach seem rather implausible in this setting. Second, in the iterative case the contextualist version of Adams' Thesis is in fact strong enough to land us into triviality results.

4.3 Edgington's expressivism

On the view advocated by Edgington (1986, 1995) (see also (Bennett, 2003)), conditionals do not express propositions. Rather, an indicative conditional $\mathsf{A} \Rightarrow \mathsf{B}$ is a device for expressing high conditional credence in B given A. The view avoids triviality results in a radical way: it holds that probabilities only attach to sentences that express proposition, and that since conditionals do not express propositions, probabilities cannot attach to them.

Looking at the frequential setting brings out two issues with this view. First, it makes clear that conditionals are at least not *always* devices to express high conditional probability. For instance, (15) can be used not only to talk about a single but unknown outcome, but also to state a generalization about a sequence of known outcomes (say, 3, 5, 2, 2, 1).

(15) If the outcome was even, it was two.

Here, the role of the *if*-clause is not to restrict the range of epistemic possibilities, but to restrict the range of relevant occasions. This shows that Edgington's view cannot be the whole story about the interpretation of indicative conditionals—at least if one wants a general account that avoids postulating ambiguity.

Second, to maintain a uniform account of probability and frequency, a proponent of this view should also deny that frequencies can be attached to conditionals. This, however, leaves one with the challenge of accounting for the relevant frequency intuitions. While in the case of probability one may perhaps exploit the link between conditionals and the expression of conditional credence to give an error theory about probability judgments, it is unclear what would lead to the structurally analogous judgments for frequencies and multiplicities.

5 Two unified accounts

What prospects are there for a unified account of probabilistic and frequential triviality? We argued in §3 that the key lesson from the frequential triviality results is to go non-propositional: probabilities and frequencies are not assigned to conditionals via propositions or occasion types. We now sketch how two non-propositional approaches to probabilistic triviality can be extended in a natural way to frequencies.[16]

5.1 A three-valued approach

One approach which identifies probabilities of conditionals with conditional probabilities while also avoiding triviality is the three-valued approach.[17] This proposal can be naturally extended to give an account of our judgments about frequencies of conditionals.

On our initial picture, a sentence is either true or false at any point of evaluation. The three-valued approach rejects this assumption. It allows that in some cases, a sentence may take a third truth value, "undefined", denoted u.

In the semantics, sentences are evaluated at pairs of worlds and occasions. The semantics is then of the form:

$$[\![\mathsf{A}]\!]^{w,o} \in \{u, 0, 1\}$$

Given a sentence A, fixing an occasion o, A does not always determine a proposition, i.e., a bipartition of the space of worlds into a truth-set and a falsity-set. Instead it determines a *partial proposition*: a tripartition of the worlds according to whether the sentence is true, false or undefined at that world (relative to o). We will let $\mathbf{A}^o_1$ be the set of worlds at which A is true (relative to o):

$$\mathbf{A}^o_1 := \{w \mid [\![A]\!]^{w,o} = 1\}$$

Further, we can consider the set of worlds at which A has a "defined" truth value, i.e., either true or false:

$$\mathbf{A}^o_d := \{w \mid [\![\mathsf{A}]\!]^{w,o} \in \{0, 1\}\}$$

The function of an *if*-clause is to selectively partialize a proposition: a conditional $\mathsf{A} \Rightarrow \mathsf{B}$ is undefined when A is false or undefined, and when A is true, it takes the truth value of B.

$$[\![\mathsf{A} \Rightarrow \mathsf{B}]\!]^{w,o} = \begin{cases} [\![\mathsf{B}]\!]^{w,o} & \text{if } [\![\mathsf{A}]\!]^{w,o} = 1 \\ u & \text{otherwise} \end{cases}$$

What is the point of partialization? One function of partialization is to put into effect a restriction in

[16] Another recent approach to probabilities of conditionals we take to be of this kind is Goldstein and Santorio (2021). We do not investigate whether that account can be extended to frequencies here.

[17] The table for the conditional is due to de Finetti (1936) and Reichenbach (1944). For more recent work in the three-valued approach in connection with probability, see (McDermott, 1996; Milne, 1997; Cantwell, 2006; Rothschild, 2013; Lassiter, 2020; Égré *et al.*, 2023). For recent work on the logical side, see Égré *et al.* (2020).

assessments of probability or frequency.[18]

Let us focus first on the case of probability. Suppose the occasion o is fixed. To improve readability, we omit reference to o and write $\mathbf{A}_1$ for $\mathbf{A}_1^o$ and $\mathbf{A}_d$ for $\mathbf{A}_d^o$. In an epistemic state s, the probability of a sentence (relative to the fixed occasion o) is the probability that the sentence is true, conditional on the sentence having a defined truth value:

$$P_s(\mathsf{A}) := p_{s+\mathbf{A}_d}(\mathbf{A}_1) = \frac{p_s(\mathbf{A}_1)}{p_s(\mathbf{A}_d)}$$

Note that for bivalent sentences, $p_s(\mathbf{A}_d) = 1$ and the definition boils down to the standard definition of probability as probability of truth.

We can still construe conditional probabilities, as we did above, as probabilities that arise from restricting the epistemic space to worlds where the condition is true:

$$P_s(\mathsf{B}|\mathsf{A}) = P_{s+\mathbf{A}_1}(\mathsf{B})$$

As a consequence, Adams' Thesis holds:

$$\begin{aligned}
P_s(\mathsf{A} \Rightarrow \mathsf{B}) &= \frac{p_s((\mathbf{A} \Rightarrow \mathbf{B})_1)}{p_s((\mathbf{A} \Rightarrow \mathbf{B})_d)} \\
&= \frac{p_s(\mathbf{A}_1 \cap \mathbf{B}_1)}{p_s(\mathbf{A}_1 \cap \mathbf{B}_d)} \\
&= \frac{p_{s+\mathbf{A}_1}(\mathbf{B}_1)}{p_{s+\mathbf{A}_1}(\mathbf{B}_d)} \\
&= P_{s+\mathbf{A}_1}(\mathsf{B}) \\
&= P_s(\mathsf{B}|\mathsf{A})
\end{aligned}$$

A completely parallel treatment is possible for frequencies. Holding fixed a world w, we have the set of occasions on which A is true, and the set of occasions at which A takes a "defined" truth value:

$$\mathcal{A}_1^w := \{o \in k \mid [\![A]\!]^{w,o} = 1\}$$

$$\mathcal{A}_d^w := \{o \in k \mid [\![\mathsf{A}]\!]^{w,o} \in \{0,1\}\}$$

Then, relative to an occasion domain k and a fixed world w (which we suppress reference to for simplicity), the frequency of a sentence A is the frequency of A being true in the set of occasions where A has a defined

[18] Another function is to provide restrictions to modals and other quantifiers: see (Belnap, 1970; Huitink, 2009; Rothschild, 2011; von Fintel and Gillies, 2015).

truth value:

$$F_k(\mathsf{A}) := f_{k+\mathcal{A}_d}(\mathcal{A}_1) = \frac{f_k(\mathcal{A}_1)}{f_k(\mathcal{A}_d)}$$

Again, for bivalent sentences we have $f_k(\mathcal{A}_d) = 1$, and the definition boils down to the previous definition of frequency.

The conditional frequency of B given A can still be construed, as above, as the frequency of B among those occasions where A is true:

$$F_k(\mathsf{B}|\mathsf{A}) = F_{k+\mathcal{A}_1}(\mathsf{B})$$

As a consequence, Adams' Thesis for frequencies holds:

$$F_k(\mathsf{A} \Rightarrow \mathsf{B}) = F_k(\mathsf{B}|\mathsf{A})$$

The proof is exactly the same as in the case of probabilities.

Thus, the three-valued approach allows us to give a unified semantics for indicative conditionals and structurally parallel accounts of probabilities and frequencies which generalize the standard ones, and which vindicate both versions of Adams' Thesis.

5.2 Attitude Semantics

Another approach to the probabilities of conditionals that can be extended to the case of frequencies is the Attitude Semantics recently developed by Ciardelli (2021, 2022). Originally, the approach was developed for the epistemic setting, where it allows us to assign probabilities to epistemic conditionals in accordance with Adams' Thesis, while avoiding triviality. In this section we briefly review the key ideas, and then sketch how they can be extended to the iterative setting so as to vindicate the frequential Adams' Thesis.

In reviewing Attitude Semantics, let us temporarily forget about occasions (and other contextual parameters of interpretation), and let us think of a sentence A such as 'the outcome is even' as being true or false relative to a world, $[\![\mathsf{A}]\!]^w \in \{0, 1\}$.

However, the story goes, not all sentences can be interpreted at this level, i.e., in terms of truth and falsity at a world; in particular, indicative conditionals of the epistemic kind cannot. Those sentences that *can* be interpreted in terms of truth and falsity at a world are called *factual* sentences. As usual, such sentences are associated with a proposition $\mathbf{A} = \{w \mid [\![\mathsf{A}]\!]^w = 1\}$.

Beyond the factual layer of language, we have a more general layer, called the *epistemic* layer, which contains, in addition to factual sentences that describe states of affairs, also sentences involving epistemic modals (such as *might*, *must*, and *probably*) and indicative conditionals of the epistemic kind. The idea is that the role of these sentences is not to describe how things are, but to express, or recommend, a certain epistemic *stance* about how things are.

The semantics that interprets the epistemic layer of language makes use of two parameters (in addition to the context parameter, which we set aside). The first is an epistemic state s, modeled as described in §2.1 as a probability space. The second parameter is called the *attitude parameter*, since it is thought of as related to the attitude expressed towards a certain propositional content. However, in view of the generalization discussed below, it seems preferable to use a more neutral term for this parameter; we will refer to it as the *prevalence* parameter, and use π as a variable for it; among the possible values of this parameter we have at least the universal prevalence ($\forall$), the existential prevalence ($\exists$) and partial prevalences (π_t) for each degree $t \in [0, 1]$.

Summing up, then, the semantics of the epistemic layer takes the form of an assignment

$$[\![\mathsf{E}]\!]^{s,\pi} \in \{0, 1\}$$

which assigns to a (factual or non-factual) sentence E a truth value relative to an epistemic state s and a prevalence π.[19] For factual sentences A, the clauses are as follows, where L_s is the set of live possibilities associated with s:

- $[\![\mathsf{A}]\!]^{s,\forall} = 1 \iff \forall w \in L_s : [\![A]\!]^w = 1$
- $[\![\mathsf{A}]\!]^{s,\exists} = 1 \iff \exists w \in L_s : [\![A]\!]^w = 1$
- $[\![\mathsf{A}]\!]^{s,\pi_t} = 1 \iff p_s(\mathbf{A}) \geq t$

In words, a factual sentence A is satisfied in s with universal prevalence if it is true in all live possibilities associated with s; it is satisfied with existential prevalence if it is true in some live possibilities; and it is satisfied with partial prevalence to degree t if the probability that s assigns to A being true is at least t.

Epistemic modals are interpreted as shifters of the prevalence parameter:

- $[\![\mathsf{must(E)}]\!]^{s,\pi} = [\![\mathsf{E}]\!]^{s,\forall}$
- $[\![\mathsf{might(E)}]\!]^{s,\pi} = [\![\mathsf{E}]\!]^{s,\exists}$
- $[\![\mathsf{prob(E)}]\!]^{s,\pi} = [\![\mathsf{E}]\!]^{s,\pi_{t^*}}$ for a fixed threshold t^*

Epistemic conditionals with factual antecedents are treated as claims about a restricted set of possibilities. Denoting the epistemic conditional construction by the notation $\Rightarrow_e$, we have:

[19] The idea is that s and π are not contextual parameters—i.e., they are not to be thought of as fixed by the context of utterance. Instead, when a sentence E is asserted, what is expressed is a certain content, modeled by the set $\{\langle s, \pi\rangle \mid [\![\mathsf{E}]\!]^{s,\pi} = 1\}$. For a discussion of the role that such contents are supposed play in communication, see (Ciardelli, 2021, 2022).

- $[\![\mathsf{A} \Rightarrow_e \mathsf{E}]\!]^{s,\pi} = [\![\mathsf{E}]\!]^{s+\mathbf{A},\pi}$

One aim of the semantics is to give a plausible account of the systematic compositional interaction between conditionals and epistemic modals. For instance, (16-a) and (16-b) are predicted to be synonymous: both sentences claim that the conditional probability of six given even is high.

(16) a. If the outcome was even, it was probably a six. $\mathsf{even} \Rightarrow_e \mathsf{prob}(\mathsf{six})$

b. Probably, if the outcome was even it was a six. $\mathsf{prob}(\mathsf{even} \Rightarrow_e \mathsf{six})$

Another aim is to give an account of probabilities that covers also probabilities of non-factual sentences. Here, the idea is to construe the probability of a sentence E is a measure of the degree to which E is supported by an epistemic state. More formally, the probability of E in a state s is the largest value $t \in [\![0, 1]\!]$ such that E is satisfied in s to degree t:

$$P_s(\mathsf{E}) = \max\{t \in [0, 1] \mid [\![\mathsf{E}]\!]^{s,\pi_t} = 1\}$$

It is immediate to check that, for factual sentences A, the probability of A obtained in this way coincides with the probability of the proposition expressed by A, i.e., with the probability that the world is one where A is true:

$$P_s(\mathsf{A}) = p_s(\mathbf{A})$$

Moreover, one can check that the probability of an epistemic conditional $\mathsf{A} \Rightarrow_e \mathsf{B}$, where A and B are factual, is the conditional probability of B given A, in accordance with Adams' Thesis:

$$P_s(\mathsf{A} \Rightarrow_e \mathsf{B}) = P_s(\mathsf{B}|\mathsf{A}) = P_{s+\mathbf{A}}(\mathsf{B}) = \frac{p_s(\mathbf{A} \cap \mathbf{B})}{p_s(\mathbf{A})}$$

Having thus briefly reviewed how the proposal works in the original setting, let us see how it can be extended to the iterative case. The starting point is to bring the occasional dimension into the picture. As we pointed out above, sentences such as 'the outcome was even' are true or false, not just relative to a world, but also relative to a specific occasion—a particular die roll. Let us call such factual sentences *local*. At the basic level, such sentences are thus assigned a truth value $[\![\mathsf{A}]\!]^{w,o} \in \{0, 1\}$ relative to a world w and an occasion o (and other contextual parameters, that we set aside).

Beyond this local layer of language, we then have a more general layer of factual sentences which comprises, in addition to local sentences, also sentences obtained from them by means of adverbs of quantification (*sometimes*, *always*, *usually*, *never*, etc.) and iterative conditionals, denoted by the notation $\Rightarrow_i$.

Semantically, the move from the local to the general layer of factual sentences is closely parallel to the move from the factual layer to the epistemic one. General factual sentences will be interpreted relative to a world w, a set of occasions k, and a prevalence parameter π. As above, we will take the range of the prevalence parameter to include at least the universal prevalence $\forall$, the existential prevalence $\exists$, and partial prevalences π_t for each $t \in [0,1]$.

If A is a local sentence, the semantic clauses are as follows, where $\mathcal{A}_w$ denotes the occasion type $\{o \mid [\![\mathsf{A}]\!]^{w,o} = 1\}$:

- $[\![\mathsf{A}]\!]^{w,k,\forall} = 1 \iff \forall o \in k : [\![A]\!]^{w} = 1$
- $[\![\mathsf{A}]\!]^{w,k,\exists} = 1 \iff \exists o \in k : [\![A]\!]^{w} = 1$
- $[\![\mathsf{A}]\!]^{w,k,\pi_t} = 1 \iff \#(k \cap \mathcal{A}_w)/\#k \geq t$

Adverbial quantifiers are treated, like epistemic modals, as shifters of the prevalence parameter, for example:

- $[\![\mathsf{always}(\mathsf{G})]\!]^{w,k,\pi} = [\![\mathsf{G}]\!]^{w,k,\forall}$
- $[\![\mathsf{stimes}(\mathsf{G})]\!]^{w,k,\pi} = [\![\mathsf{G}]\!]^{w,k,\exists}$
- $[\![\mathsf{usually}(\mathsf{G})]\!]^{w,k,\pi} = [\![\mathsf{G}]\!]^{w,k,\pi_{t_0}}$ for a fixed threshold t_0

Indicative conditionals of the iterative kind are treated as claims about a restricted set of occasions:

- $[\![\mathsf{A} \Rightarrow_i \mathsf{G}]\!]^{w,k,\pi} = [\![\mathsf{G}]\!]^{w,k+\mathcal{A}_w,\pi}$

Mirroring the epistemic case, a benefit of the theory is that it gives a plausible account of the interaction between conditionals and adverbs of quantification. For example, the following two statements are predicted to be equivalent: both claim, roughly, that most (relevant) occasions where the weather is nice are occasions where George has breakfast in the garden.

(17) a. If the weather is nice, George usually has breakfast in the garden.

b. Usually, if the weather is nice George has breakfast in the garden.

This is the analysis famously advocated by Lewis (1975). However, unlike in Lewis' account, this result is obtained not by treating the conditional and the adverbial quantifier as part of a single construction, but compositionally, assigning separate contributions to these items.

In this setting, we can give a treatment of frequencies which parallels the above treatment of probabilities. The frequency of G within a set of occasions k (relative to world w) is the largest t such that G is true in k with prevalence π_t. More formally:

$$F_{w,k}(\mathsf{G}) = \max\{t \in [0,1] \mid [\![\mathsf{G}]\!]^{w,k,\pi_t} = 1\}$$

The results that we discussed for probabilities can be replicated straightforwardly. First, if A is a local factual sentence, then the frequency of A in k is just the frequency of the corresponding occasion type $\mathcal{A}_w$, i.e., the proportion of occasions on which A is true to the total number of occasions, as per our original assumptions:

$$F_{w,k}(\mathsf{A}) = f_k(\mathcal{A}_w) = \frac{\#(k \cap \mathcal{A}_w)}{\#k}$$

Second, if A and B are local factual sentences, we can show that the frequency of the iterative conditional $\mathsf{A} \Rightarrow_i \mathsf{B}$ is the conditional frequency of B given A, in accordance with Adams' Thesis for frequencies:

$$F_{w,k}(\mathsf{A} \Rightarrow_i \mathsf{B}) = F_{w,k}(\mathsf{B}|\mathsf{A}) = F_{w,k+\mathcal{A}_w}(\mathsf{B}) = \frac{\#(k \cap \mathcal{A}_w \cap \mathcal{B}_w)}{\#(k \cap \mathcal{A}_w)}$$

Thus, the ideas of Attitude Semantics can be exported to the iterative setting in such a way as to get structurally parallel accounts of probabilities and frequencies, which coincide with the standard ones for run-of-the-mill sentences and which vindicate both versions of Adams' Thesis without triviality.

5.3 Back to the lesson

Our diagnosis about the frequential triviality problem was that it stems from the assumption that frequencies and multiplicities are always frequencies and multiplicities of occasion types. By analogy, we concluded that the source of the probabilistic triviality problem is the assumption that probabilities are always probabilities of propositions.

The two approaches in this section allow for a unified account of triviality by heeding this lesson: in both approaches, probabilities and frequencies are sometimes, but not always, probabilities of corresponding propositions or occasion types. Instead, probabilities attach to a more general kind of semantic object, which may or may not correspond to a standard proposition or occasion type. For instance, in the three-valued approach, the bearers of probability are *partial* propositions, and the bearers of frequency are *partial* occasion types.

Both approaches also agree with our diagnosis of the fallacy in Lewisian triviality proofs, in that probabilities and frequencies of conditionals do not behave standardly. In particular, both approaches invalidate the law of total probability for conditionals, both for probabilities and frequencies. For example, the judgments from our counterexample in §3 are predicted in either case.

These accounts also explain why the usual laws of probability theory can be generally satisfied for non-conditional sentences, but fail to generalize specifically to conditionals. In the special case where a sentence has a defined truth value in every world or on any occasion, as may often be for non-conditional sentences, probabilities and frequencies behave standardly.

6 Conclusion

The epistemic and iterative readings of indicative conditionals share a striking list of formal analogies. Comparing judgments of probabilities and frequencies of conditionals suggests this list should include Adams' Thesis, on the one hand for probabilities, on the other for frequencies. As a consequence, the problem of triviality results arises also in the iterative setting.

We believe that appreciating the iterative side of the triviality problem can be illuminating. This is in large part due to occasions and frequencies being, in some ways, importantly different from and less mysterious than epistemic possibilities and probabilities. We have argued that the new results motivate a series of conclusions about what the source of the problem is, and what it is not. The triviality problem is not a problem about rational constraints on credence, or about assertibility, the representation of epistemic states, or equivocation due to context shifts. Rather, it is a problem having to do with the compositional contribution of *if*-clauses and the peculiar contents expressed by conditionals, which arises in much the same way in a non-epistemic setting as in an epistemic setting.

What the iterative case shows is that frequencies of conditionals are not frequencies of standard occasion types. This strongly suggests that also probabilities of conditionals are not probabilities of standard propositions.

As a consequence, we should not expect probabilities and frequencies of conditionals to behave in the same way as "standard" probabilities and frequencies arising from a proposition or an occasion type. Indeed, we argued that the law of total probability fails for frequencies of conditionals.

Probabilities and frequencies of conditionals seem instead to be probabilities and frequencies that result from the antecedent restricting the relevant domain. On this diagnosis, there still remains room for disagreement about how this restricting role is played compositionally, for example by partializing a proposition (as in three-valued theories), or by targeting a domain parameter that the semantics is sensitive to (as in attitude semantics). The two specific options we considered by no means exhaust the space of possibilities. Our aim in considering them is primarily to show how our main conclusions could be made concrete in specific accounts of the semantics of conditionals and of the notions of probability and frequency.

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