The Difference between Knowledge and Understanding

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KNOWLEDGE = JUSTIFIED TRUE BELIEF.
Smith believes from experience

\[ q \quad \text{... Jones owns a Ford.} \]

and also believes

\[ p \quad \text{... Someone in the office owns a Ford.} \]
Gettier case

$q \ldots \text{Jones owns a Ford}$

$\downarrow$

$p \ldots \text{Someone in the office owns a Ford.}$

$\downarrow$

*justified belief* in $p$
Gettier case

\[ q = \text{Jones owns a Ford.} \quad \text{false} \]

\[ \Downarrow \]

\[ p = \text{Someone in the office owns a Ford.} \]
Gettier case

\[ q = \text{Jones owns a Ford.} \quad \text{false} \]

\[ \Downarrow \]

\[ p = \text{Someone in the office owns a Ford.} \quad \text{true} \]
Gettier case

\[ q = \text{Jones owns a Ford.} \quad \text{false} \]

\[ p = \text{Someone in the office owns a Ford.} \quad \text{true} \]

\[ r = \text{Brown owns a Ford.} \quad \text{true} \]
Gettier case

$q = \text{Jones owns a Ford.} \quad \text{false}$

$p = \text{Someone in the office owns a Ford.} \quad \text{true}$

$r = \text{Brown owns a Ford.} \quad \text{true}$

... oops
Gettier case

$q = \text{Jones owns a Ford.} \quad \text{false}$

$\Downarrow$

$p = \text{Someone in the office owns a Ford.} \quad \text{true}$

$r = \text{Brown owns a Ford.} \quad \text{true}$

... justified, true belief in $p$

but not knowledge
Improve on the hamster wheel

Consider a more complete training regimen for your pet
IMPROVE ON THE HAMSTER WHEEL

CONSIDER A MORE COMPLETE TRAINING REGIMEN FOR YOUR PET
Plan

1. Added value of knowledge over true belief follows from the tracking conditions.

2. Tracking improves relevance matching, hence Gettierization avoidance (w/o ad hoc additions).

3. Don’t need to presuppose value of knowledge to see value of gettierization avoidance.

4. Understanding $\approx$ relevance matching.

5. Understanding is simulation.
The True Belief Game – Approx.

Payoff assumptions: $p$ true $\rightarrow$ (believe $>$ not believe), $p$ false $\rightarrow$ (not believe $>$ believe)
“Mere” good and bad states

Good belief states:

- $p$ true  $S$ believes $p$  true belief
- $p$ false  $S$ does not believe $p$  good lack of belief

Bad belief states:

- $p$ true  $S$ does not believe $p$  bad lack of belief
- $p$ false  $S$ believes $p$  false belief
“Mere” good and bad states

**Good belief states:**

<table>
<thead>
<tr>
<th>p true</th>
<th>S believes p</th>
<th>true belief</th>
</tr>
</thead>
<tbody>
<tr>
<td>p false</td>
<td>S does not believe p</td>
<td>good lack of belief</td>
</tr>
</tbody>
</table>

**Bad belief states:**

<table>
<thead>
<tr>
<th>p true</th>
<th>S does not believe p</th>
<th>bad lack of belief</th>
</tr>
</thead>
<tbody>
<tr>
<td>p false</td>
<td>S believes p</td>
<td>false belief</td>
</tr>
<tr>
<td>Belief state:</td>
<td>$p$ true, $S$ doesn’t believe $p$</td>
<td></td>
</tr>
<tr>
<td>--------------</td>
<td>----------------------------------</td>
<td></td>
</tr>
<tr>
<td>Strategy:</td>
<td>In response to $p$, don’t believe $p$</td>
<td></td>
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<tr>
<td></td>
<td>In response to $\neg p$, don’t believe $p$</td>
<td></td>
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<tr>
<td></td>
<td><em>(disposition, regularity)</em></td>
<td></td>
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</tbody>
</table>
The True Belief Game – Approx.

Payoff assumptions: p true → (believe > not believe),
                   p false → (not believe > believe)
Belief state vs. Strategy

Belief state: $p$ true, $S$ doesn’t believe $p$

Strategy: In response to $p$, don’t believe $p$
In response to $\neg p$, don’t believe $p$

*disposition, rule* for responding to all possible plays of opponent.
Belief state vs. Strategy

Belief state:  $p$ true, $S$ doesn’t believe $p$

  $p, -b(p)$

Strategy:  disposition, regularity for responding to all possible plays of opponent.

  e.g. Tracking is a strategy:

1)  $P(-b(p)/-p) > s$

2)  $P(b(p)/p) > t$
Knowledge = Tracking

Tracking is a strategy:

1) $P(-b(p)/-p) > s$

2) $P(b(p)/p) > t$

Variation (Sensitivity)

Adherence
The True Belief Game – Approx.

<table>
<thead>
<tr>
<th></th>
<th>b(p)</th>
<th>- b(p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>(0,10)</td>
<td>(0,-20)</td>
</tr>
<tr>
<td>- p</td>
<td>(0,-7)</td>
<td>(0,5)</td>
</tr>
</tbody>
</table>

Payoff assumptions: p true → (believe > not believe),
                   p false → (not believe > believe)
The subject who is a tracker of p has an

**Evolutionarily Stable Strategy (ESS)**
Tracker is evolutionarily stable

→ Tracking type (R) **strictly dominates** any type following any other conditions beyond true belief (-R), in the struggle for survival and utiles.

→ Once this strategy is achieved by some level of majority of the population, no small population with an alternative strategy can “invade” and drive it out.

→ These properties hold independently of the dynamics of interaction.
If we think intuitively that knowledge can be of evolutionary or utilitarian value, then this is a unique explanatory advantage of the tracking theory.

This shows (tracking) knowledge is more valuable than mere true belief, without ad hoc tinkering.
p = Route A will get me to Larissa by 12.

Suppose:

\[ p \text{ is true} \]

\[ S, S' \text{ believe } p \]

S uses a paper map. \hspace{1cm} S' uses real-time GPS.
p = Route A will get me to Larissa by 12.

p is true
S, S’ believe p
S’ has a strong disposition to believe p when it’s true and not believe p when it’s false.

S uses a paper map.  
S’ uses real-time GPS.

S has a true belief.  
S’ has a true belief and is tracking.
p = Route A will get me to Larissa by 12.

p is true
S, S’ believe p
S’ has a strong disposition to believe p when it’s true and not believe p when it’s false.

S uses a paper map. S’ uses real-time GPS.

S has a true belief. S’ has a true belief and a contingency detector.
The Gettier Problem
Gettier cases and relevance

\[ p = \text{Someone in the office owns a Ford.} \quad true \]
\[ q = \text{Jones owns a Ford.} \quad false \]
\[ r = \text{Brown owns a Ford.} \quad true \]
Gettier cases and relevance

\[ p = \text{Someone in the office owns a Ford.} \quad true \]

\[ q = \text{Jones owns a Ford.} \quad false \]

\[ r = \text{Brown owns a Ford.} \quad true \]

\[ P(b(p)/-q.r) = P(b(p)/-q.-r) \]

but

\[ P(p/-q.r) \neq P(p/-q.-r) \]
q is (positively) relevant to your believing p.

\[ P(b(p)/q) \gg P(b(p)/-q) \]

Or:\n\[ \frac{P(b(p)/q)}{P(b(p)/-q)} \gg 1 \]
q is (positively) relevant to p

\[ P(p/q) \gg P(p/-q) \]

Or:

\[ \frac{P(p/q)}{P(p/-q)} \gg 1 \]
**Relevance matching** on q for p:

\[
P(b(p)/q)/P(b(p)/-q) = P(p/q)/P(p/-q)
\]

The difference q’s truth value makes to your *belief* in p is the same as the difference q’s truth value makes to p’s truth value.

**Relevance mismatch** on q for p

\[
P(b(p)/q)/P(b(p)/-q) \neq P(p/q)/P(p/-q)
\]

q’s truth value makes more of a difference, or less of a difference, to your *belief* in p than it does to p’s truth value.
Gettier case

\[ p = \text{Someone in the office owns a Ford.} \quad \text{true} \]
\[ q = \text{Jones owns a Ford.} \quad \text{false} \]
\[ r = \text{Brown owns a Ford.} \quad \text{true} \]

\[ P(b(p)/q) \gg P(b(p)/\neg q) \]

but

\[ P(p/q) > P(p/\neg q) \]
**Relevance matching** on q for p:
\[ \frac{P(b(p)/q)}{P(b(p)/-q)} = \frac{P(p/q)}{P(p/-q)} \]

**Relevance mismatch** on q for p
\[ \frac{P(b(p)/q)}{P(b(p)/-q)} \neq \frac{P(p/q)}{P(p/-q)} \]

**Gettierization** → relevance mismatch for p on some q for which \( P(b(p)/q) >> P(b(p)/-q) \)

or ...

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Relevance matching on q for p:

\[ \frac{P(b(p)/q)}{P(b(p)/-q)} = \frac{P(p/q)}{P(p/-q)} \]

Relevance mismatch on q for p

\[ \frac{P(b(p)/q)}{P(b(p)/-q)} \neq \frac{P(p/q)}{P(p/-q)} \]

Gettierization → relevance mismatch for p on some r for which \( P(p/r) \gg P(p/-r) \)
Gettierized belief in $p$

Depends on:

1) basing belief in $p$ on $q$ (the helper) when $q$ is false
2) having a relevance mismatch on $q$ for 1) to exploit
3) $p$ is true
Relation of *Relevance Matching* for $p$ and *Tracking* $p$

\[
P(b(p)/q) = P(b(p)/p)P(q/b(p).p)P(p/q) + P(q/p)\]
\[
P(b(p)/-p)P(q/b(p).-p)P(-p/q)\]
\[
P(q/-p)\]

\[
P(b(p)/-q) = P(b(p)/p)P(-q/b(p).p)P(p/-q) + P(-q/p)\]
\[
P(b(p)/-p)P(-q/b(p).-p)P(-p/-q)\]
\[
P(-q/-p)\]
Relevance Matching

\[
\frac{P(b(p)/q)}{P(b(p)/-q)} = \frac{P(p/q)}{P(p/-q)}
\]
Relation of *Relevance Matching* for $p$ and *Tracking* $p$

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P(b(p)/q) = P(b(p)/p)P(q/b(p).p)P(p/q) + \]
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P(q/p) \]
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P(b(p)/-p)P(q/b(p).-p)P(-p/q) + \]
\[
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\[
P(b(p)/-q) = P(b(p)/p)P(-q/b(p).p)P(p/-q) + \]
\[
P(-q/p) \]
\[
P(b(p)/-p)P(-q/b(p).-p)P(-p/-q) + \]
\[
P(-q/-p) \]
Perfect Sensitivity to p

\[ P(b(p)/q) = P(b(p)/p)P(q/b(p).p)P(p/q)P(q/p) \]

\[ P(b(p)/-q) = P(b(p)/p)P(-q/b(p).p)P(p/-q)P(-q/p) \]
Relation of *Tracking* $p$ to

*Relevance Matching* for $p$ on $q$

\[ P(b(p)/q) = \alpha \ P(p/q) \]

\[ P(b(p)/-q) = \alpha \ P(p/-q) \]
Relation of Tracking $p$ to Relevance Matching for $p$

\[
P(b(p)/q) = P(p/q) = P(b(p)/-q) = P(p/-q)
\]
Relation of *Tracking* p to *Relevance Matching* for p

\[
P(b(p)/q) = P(p/q)
\]

\[
P(b(p)/-q) = P(p/-q)
\]

1. Perfect tracking of p ⇒ Perfect relevance matching for p on q
Relation of *Tracking* p to *Relevance Matching* for p

\[
\begin{align*}
P(b(p)/q) & \quad = \quad P(p/q) \\
P(b(p)/-q) & \quad = \quad P(p/-q)
\end{align*}
\]

1. Perfect tracking of p \(\Rightarrow\) Perfect relevance matching for p on q, *for all* q

I.e., perfect tracking \(\Rightarrow\) No possibility of gettierization (on any q)
Relation of *Tracking* \( p \) to *Relevance Matching* for \( p \)

\[
\begin{align*}
\frac{P(b(p)/q)}{P(b(p)/-q)} &= \frac{P(p/q)}{P(p/-q)}
\end{align*}
\]

1. Perfect tracking of \( p \) \( \Rightarrow \) Perfect relevance matching for \( p \) on \( q \), *for all* \( q \)

2. Increased tracking \( \Rightarrow \) Increased relevance matching for \( p \) on every \( q \)
Relation of *Tracking p* to *Relevance Matching* for p

\[
\frac{P(b(p)/q)}{P(b(p)/-q)} = \frac{P(p/q)}{P(p/-q)}
\]

1. Perfect tracking of p ⇔ Perfect relevance matching for p on all q

2. Increased tracking of p ⇒ Increased relevance matching for p on all q

3. Increased relevance matching for p on a given q ☓ Increased tracking of p
Perfect tracking

p \leftrightarrow b(p)

Perfect relevance matching
p \rightarrow q \rightarrow b(p) \rightarrow p

p \rightarrow q \rightarrow b(p) \rightarrow p
Gettier cases, relevance matching, and understanding

\[ p = \text{someone in the office owns a Ford.} \]
\[ q = \text{Jones owns a Ford.} \]
\[ r = \text{Brown owns a Ford.} \]

Have: \( P(p/q) = 1, P(b(p)/q) = 1 \)

But: \( P(p/-q) \neq P(b(p)/-q) \)

Other ways than \( q \) of making \( p \) true are more relevant to \( p \) than \( S \)'s belief dispositions reflect.

\( S \) doesn’t understand why \( p \) is true.
Definition – first pass

If S believes p and p is true, then

S’s *understanding* of why p is true *improves* iff there is an increase in relevance matching for p on some q and no outweighing decrease in relevance matching for other q.
Recall

Increasing your tracking of $p$ will increase your relevance matching for $p$ on every $q$.

$\implies$ Tracking brings relevance matching, G-avoidance, and understanding.

Increasing your relevance matching on a given $q$ doesn’t necessarily increase your tracking of $p$. 
Knowledge and Understanding

Increasing your tracking of p will increase your relevance matching for p on every q.

→ Knowledge brings relevance matching, G-avoidance, and understanding.

Increasing your relevance matching on a given q doesn’t necessarily increase your tracking of p.

But improved understanding of p always improves level of tracking (knowledge) of p.
Understanding and Explanation

**Fact:** Relevance matching your belief in p to the web of q’s relevant to p does not require you to be able to cite the factors probabilistically relevant to p.

**Opinions:**

1. If we add a citation requirement, then we get a definition of ability to give an explanation. (= Salmon statistical relevance view)
2. Not all understanding brings ability to give explanations.
Prediction of human behavior

S: I know what she’ll do.
A: How do you know?
S: I understand her.

We do this without being able to list all the factors. (Challenge for the higher-order view of understanding other minds.)
p’s web of relevance
Mere true belief in $p$

Diagram:

- $q_1$
- $q_2$
- $q_3$
- $q_4$
- $b(p)$
- $p$
- $q_5$
- $q_6$
- $q_7$
- $q_8$
Relevance Matching, Understanding?
Understanding

Understanding *why you should believe* \( p \)

Understanding *why* \( p \) is true
Understanding

\[ p = \text{Jefferson is dead} \]

Understanding \textit{why you should believe} \( p \)
\[ q_1 = \text{lack of pulse} \]

Understanding \textit{why} \( p \)
\[ q_2 = \text{gunshot wound} \]
\[ q_3 = \text{political disputes} \]

\textit{indicators of} \( p \) \hspace{1cm} \textit{vs.} \hspace{1cm} \textit{what makes} \( p \) \text{ true}
Awkward

You track $p$ via a great indicator

$\Rightarrow$ You relevance match on all $q$.

$\Rightarrow$ You understand why Jefferson is dead.
Your believing $p$ (hurricane tomorrow) co-varies with output of a great computer simulation programmed by someone else.

⇒ You track $p$.
⇒ You relevance match on all $q$.
⇒ You understand why $p$ is true.
Hyperbolic Intellectualism
Understanding?
Owning the relevance matching
Prediction of human behavior

S: I know what she’ll do.
A: How do you know?
S: I understand her.

We do this without being able to list all the factors. (Challenge for the higher-order view of understanding other minds.)
Understanding as simulation

\[ m(q_1) \quad q_2 \quad m(q_2) \quad m(q_3) \]
\[ m(q_4) \quad q_6 \quad b(p) \quad m(q_6) \]
\[ m(q_5) \quad m(q_7) \quad q_8 \quad m(q_8) \]
Summary

1. Knowledge (tracking) is more valuable than mere true belief; it is an ESS.

2. What explains that value (tracking) also directly opposes gettierization.

3. Gettierization avoidance for \( p \) has a value – contributing to understanding \( p \) – even if we don’t assume knowledge of \( p \) has value.

4. Understanding \( \sim \) relevance matching \( \sim \) simulation
Somewhere, something incredible is waiting to be known.

-Carl Sagan-
p \longleftrightarrow b(p)