

Apostolos Doxiadis, Christos H. Papadimitriou, Alecos Papadatos, and Annie di Donna, *Logicomix*, Bloomsbury USA, New York, 2009.

Essay Review by Paolo Mancosu, U.C. Berkeley

*Logicomix* is a graphic novel about Bertrand Russell's life (up to 1939) and the intellectual contribution Russell made to the foundations of logic and mathematics. Its ambition is to convey to the reader the excitement of an intellectual quest, that of the attempt to give a definitive foundation of mathematics, and the psychological complexities of the relationship between the man –his passions, his deep fears etc. – and his intellectual work.

*Logicomix* is divided into 6 main chapters to which one must add three other chapters (*Overture*, *Entracte*, *Finale*) in which the team who wrote and designed *Logicomix* talk about the project, its motivation and their, at times conflicting, takes on what the moral of the story is supposed to be. *Logicomix* also contains a *Notebook* which provides additional information on facts and ideas.

The six chapters are titled:

- 1) *Pembroke Lodge* [Russell's childhood]
- 2) *The Sorcerer's Apprentice* [Russell's studies in Cambridge and his love of Alys]
- 3) *Wanderjahre* [travels to Paris and Germany]
- 4) *Paradoxes* [Russell's paradox and collaboration with Whitehead in writing *Principia Mathematica*]
- 5) *Logico-Philosophical Wars* [relation to Wittgenstein; World War I, Russell and Wittgenstein during the war; the *Tractatus*]
- 6) *Incompleteness* [Gödel's theorems; Russell's second wife Dora, and the education of their son John (Beacon school); Wittgenstein as a teacher; neopositivism; the rise of Nazism; the issue of U.S. involvement in World War II]

The three additional chapters introduce some of the major issues that the story is also supposed to capture. For instance in *Overture* the theme of logic versus madness is introduced in a conversation between Christos and Apostolos; the topic is pursued later in other chapters and in the *Entracte* where Anne and Christos also happen to be looking for a theatre where Anne is rehearsing in a representation of Aeschilus' *Oresteia*. The introduction of tragedy is supposed to have parallels with the search for foundations and this is pursued in the *Finale*. I will come back to these themes later. But before getting there, I would like to present in more detail some biographical elements of Russell's life (section 2) and some major milestones in the foundational debate (section 3), which might help the reader in setting *Logicomix* in context. The details will also be instrumental for the critical comments in section 3.

**1. Russell's life.** *Logicomix* recounts the main events of Russell's life by following the (imagined) delivery of a lecture by Russell himself recapturing the main events of his life and work. Actually, the events of Russell's life being recounted do not go beyond those related to his early life, adolescence, student days and up to his first marriage and imprisonment for protesting the involvement of the USA in the first world war. Treated in much less detail is the period 1918-1939 where most of the attention is devoted to conceptual developments. Since the imagined lecture is delivered before the official entrance of the U.S. in War World II, no mention is made of Russell's life between 1940 and 1970.

Russell was born on 18 May 1872 from a prominent aristocratic family.<sup>(1)</sup> His paternal grandfather, John Russell, had been Prime Minister twice (1846-1852 and 1865-1866).

His parents, John and Katherine Louisa, were unconventional. His father was an atheist and accepted his wife's love relationship with Douglas Spalding, a tutor for the children.

Russell had one older brother, Frank (he does not appear in *Logicomix*), and a younger sister, Rachel. Tragedy struck the family in 1874 when both Russell's mother and his sister Rachel died of diphtheria. Two years later Russell's father died of bronchitis.

Thus, from 1876, Frank and Bertrand lived with their grandparents at Pembroke Lodge in Richmond Park. Once the grandfather died in 1878, the grandmother came to play a central role in their lives. Her religious devotion and formality led to a stern atmosphere in which Bertrand kept his inner life to himself. As a consequence, Russell grew up feeling lonely and indeed the first volume of Monk's biography of Russell is subtitled "The spirit of solitude". He thought about suicide several times and in his autobiography he claims that the desire to learn more mathematics is what kept him from his self-destructive tendencies. While he received his education from several tutors at home, it was his brother Frank who introduced him to Euclid's *Elements* and that was to have a fundamental impact in his intellectual development.

From that moment on mathematics played a central role in Russell's life and career. In 1890 he won a scholarship to read for the Mathematical Tripos at Trinity College where he studied mathematics and philosophy obtaining his B.A. in mathematics in 1893. While a student there he became close to A. N. Whitehead and G.E. Moore.

At the age of 17, Russell fell in love with Alys Pearsall Smith. They married in 1894 but it was not a happy marriage and things fell apart almost immediately. But it was only in 1910 that they separated. During the early 1910s, Russell had some significant sentimental relationships, the most important of which was the one with Ottoline Morrell (this however is not mentioned in the story). The years between 1900 and 1919 are of course central to his intellectual career and his meeting with Wittgenstein took place in 1911. But before touching on this period I will recall the last event that plays an important role in *Logicomix*, namely Russell's pacifist engagement during the WWI which led to his dismissal in 1916 from Trinity and to a later jail term of six months in Brixton prison in 1918. Russell was 46 years old at the time and *Logicomix* does not go in great

bibliographical detail beyond this part of his biography. One plausible reason for stopping there is that by this time his contribution to the foundations of mathematics was over.

## **2. Russell's intellectual contribution to the foundations of mathematics.**

In order to understand Russell's role in the foundations of mathematics, it is useful to recall Dedekind's contribution. Dedekind gave, by means of set-theoretical techniques, a thoroughgoing justification of analysis, and thus of irrational numbers, in his booklet *Continuity and Irrational Numbers* (1872). In this work irrational numbers are defined as entities corresponding to the cuts in the field of rational numbers. The reader not acquainted with cuts can visualize a cut as the set of rational numbers which are less than or equal to a given rational number. Dedekind's justification of the notion of irrational numbers presupposed however the notion of rational number and that of infinite set or rationals. It was also Dedekind's belief that the notion of number in general could be characterized by appealing to basic logical concepts. This he attempted to show in his work *Was sind und was sollen die Zahlen* (1888) which presents a foundation of the natural numbers based on his theory of chains, that is sets with specific properties. The reduction of analysis to logic (containing a large amount of what we classify as set theory) seemed to have been achieved once and for all. However, problems began to emerge. The process of reduction of arithmetic to logic had in fact used at various stages a number of problematic notions, or at least as problematic as the notions that had to be grounded, e.g. the notion of infinite set, the notion of set of all objects of thought (this appears in Dedekind's proof of the existence of an infinite set), and a number of problematic procedures, the so-called impredicative definitions (for instance defining the natural numbers as the smallest set containing every set containing 0 and closed under successor). What is characteristic of such definitions is that they define an entity (a set in the case of the natural numbers) by quantifying over a collection which already contains the entity being defined. From a constructivist point of view the definition 'generates' the entity and thus the entity being generated cannot already be part of the collection over which one quantifies in order to bring it about.

Well known is also Frege's attempt to provide a logicistic foundation for arithmetic and the great difficulties which he encountered in carrying out the project. In the case of Frege, the logicism in question is much more sharply defined than it was in Dedekind. The idea was to isolate the principles of formal logic and then, by means of a translation of mathematical concepts into logical concepts, to prove within logic the (translation of) the standard mathematical theorems. To accomplish his goal, Frege had assumed that for any property  $P(x)$  it made sense to talk about the course of values of  $P(x)$  as a

totality. More formally, and anachronistically, Frege postulated that given any  $P(x)$ , one could speak of the totality of objects satisfying  $P(x)$ , that is

$$\exists X (x \in X \text{ iff } P(x))$$

Russell's paradox showed that even at this very basic level one could run into problems. He considered the following property  $P(x) = x \notin x$  and showed that the set  $X$  such that  $x \in X \text{ iff } x \notin x$  (which is supposed to exist according to Frege's postulation) gives rise to an antinomy, that is  $X \in X \text{ iff } X \notin X$ . This effectively brought Frege's attempt to the ground.

One more central development needs to be mentioned, namely the development of set theory, due to Georg Cantor, during the last quarter of the nineteenth century. While set-theoretical procedures had already been in use, it was only with Cantor that set theory as an independent area of mathematics was born and systematized. Cantor developed ordinal and cardinal arithmetic making use of very powerful non-constructive reasoning principles and assumptions. He had also realized the danger of paradox involved in certain set-theoretic procedures and had distinguished, in correspondence with other mathematicians, between 'consistent' and 'inconsistent' totalities.

The years following the discovery of Russell's and other paradoxes witnessed an attempt to take care of them by means of different strategies. The most important ones are those of Zermelo and Russell.

Zermelo, following Hilbert's axiomatization of geometry, offered an axiomatization of set theory, which assumed only the existence of those sets whose definition could be given through a "definite propositional function" [definite Klassenaussage]. His axiom of separation was the cornerstone of the building. It stated that given any set  $Y$ , one could collect into a set  $X$  the elements of  $Y$  satisfying a property  $P(x)$ . More formally,

if  $P(x)$  is a "definite propositional function" and  $Y$  is an already given set then

$$\exists X (x \in X : x \in Y \text{ and } P(x)) \text{ [Axiom of separation].}$$

Zermelo's formalization is at the basis of our formalization of set theory (known as ZF). Notice that among the "definite propositional functions" one allows  $\neg(x \in x)$ , which is therefore a significant expression. The paradox is blocked by the fact that we can now only form, for any set  $A$ , the set  $B = \{x : x \in A \text{ and } \neg(x \in x)\}$ . But  $B$  does not give rise to a paradox.

Russell, in an attempt to revive logicism, developed a theory of types in which the type of self-referential situation evidenced by Russell's paradox could not arise. In particular, so-called impredicative definitions (see above) were excluded by eliminating even the possibility of expressing  $x \in x$  and its negation.

One of the consequences of this was a rather awkward reconstruction of mathematics. In particular one had to deal with real numbers of different levels. In the attempt to avoid these undesired consequences Russell introduced the notorious Axiom of Reducibility which states that for any set defined at some level  $n$  there is already an extensionally equivalent set at level

1. Russell's definitive achievement in this area is the three volume work *Principia Mathematica*, written with Whitehead and published in 1910-1913.

These developments were only the beginning of a some major developments in the foundations of mathematics that characterized the first three decades of the twentieth century. Indeed, among the various ways to deal with the problems generated by the new set-theoretic mathematics, and in particular the paradoxes, we find reactions such as those of Poincaré who rejected the construction of sets requiring quantification over the totality of sets to which the defined set belongs (impredicative definitions). His 'intuitionistic' foundation of mathematics was the harbinger of a more radical form of intuitionism championed by Brouwer and his intuitionist followers. To be consistent with their intuitionistic principles, these thinkers were willing to sacrifice a good deal of classical mathematics (including great parts of set theory). By contrast, Hilbert was interested in proving the consistency of all of classical number theory, analysis, and set theory. In 1905 he conceived of the possibility of treating mathematical proofs as mathematical objects. This was to lead to Hilbert's program – developed in the 1920s and 1930s -- a program for the foundation of mathematics that aimed at preserving all of mathematics by providing an axiomatization of various areas of mathematics (number theory, analysis, set theory) and proving mathematically their consistency using only mathematical principles satisfying stringent criteria of intuitiveness and evidence. This developed into the discipline of metamathematics, or proof theory, that was supposed to use only 'finitistic' thought.

Let us conclude this section then by emphasizing that the Russellian type-theoretic reconstruction of logic provided the context for technical developments in mathematical logic in the 1910s and 1920s and additionally was at the center of fundamental reflections in philosophy of mathematics, such as those carried out by Wittgenstein, Ramsey and Carnap. Moreover, an important mathematical result was to influence the course of both types of investigations: Gödel's incompleteness theorems. The theorem had profound consequences for both logicism and Hilbert's program.

*Logicomix* manages to recount much of the developments described in sections 1 and 2 with flair and appealing graphics. While the details of the technical developments could only be hinted at, the graphic novel does a splendid job at conveying the main ideas and milestones of the developments sketched both in section 1 and 2.

### 3. Critical remarks

Page 315 of *Logicomix* contains the following disclaimer:

*Logicomix* and reality: *Logicomix* was inspired by the story of the quest for the foundations of mathematics, whose most intense phase lasted from the last decades of the 19<sup>th</sup> century to the eruption of the second world war. Yet, despite the fact that its characters are mostly real persons, our book is definitely not—nor

does it want to be—a work of history. It is – and wants to be—a graphic novel.

After admitting that several aspects of the graphic novel deviate from fact then the authors add:

Still, we must add this: apart from the simplification that was necessary to accommodate it into a narrative work of this kind, we have not taken any liberties with the content of the great adventure of ideas which forms our main plot, neither with its central vision, its concepts, nor –even more importantly –with the philosophical, existential and emotional struggles that are inextricably bound with it (p. 316)

In this last section I would like to comment on

- 1) The relationship between *Logicomix* and reality;
- 2) The issue of faithfulness to the development of ideas;
- 3) Some logical inaccuracies;
- 4) Madness and tragedy.

### 3.1. *Logicomix* and reality.

The authors of *Logicomix* appeal to ‘comic license’ (p. 77) to take liberties with the real historical course of events. Some deviations are quite small and innocent. For instance, we know that it was Russell’s brother Frank who introduced him to Euclid’s *Elements* whereas in *Logicomix* this is left to a tutor and Frank does not appear. Other discrepancies are more serious. For instance, the story recounts a meeting of Russell with Cantor and one with Frege, which, as the authors frankly admit (p.315), never took place.

Of great import for the story is the description of Russell’s fears and nightmares as a child. The chapter titled “Pembroke Lodge” has Russell hearing screams coming from a secret room but no one is willing to acknowledge what is going on until he discovers that uncle Willie, his father’s brother, is secretly kept in one of the rooms and that he is mad. In actual fact, Russell never heard of uncle Willie until he was 21, namely when he decided to marry Alys and his grandmother tried everything to stop him. In particular, she revealed to him the streak of madness running through the Russell’s family. I found this deviation from reality to serve a narrative purpose by giving a vivid representation of the deep fears described by Russell when he wrote about his childhood and puberty.

By contrast, I was unable to see what role it served to describe Frege as a lunatic and a rabid anti-semitic.

[Picture 1]



While it is true that in the last two months of his life Frege wrote some rather objectionable entries in his diary (Gabriel and Kienzler 1994, Mendelsohn 1996), the caricature we are presented with seems to me to be gratuitous. The *Notebook* at the end of *Logicomix* can at times also be misleading in that while it is meant to set the historical record straight it ends up falsifying the historical record. For instance, in the entry on Frege we read:

In the last decades of his life he became increasingly paranoid, writing a series of rabid treatises attacking parliamentary democracy, labour unions, foreigners and, especially, the Jews, even suggesting “final solutions” to the “Jewish problem”. He died in 1925. (p. 326)

This obviously refers to Frege’s diary. This diary was written in the last two months of Frege’s life and not “in the last decades of his life”. Moreover, where are the “treatises”? There are three entries with comments on the Jews and although Frege’s comments on the matter are certainly objectionable, these entries do not speak of “final solutions” nor of “the Jewish problem”. This type of language was instead typical of the Nazis and by associating Frege with these expressions the *Notebook* ends up portraying Frege as a Nazi. Moreover, a reading of the notebooks does not show a paranoid or rabid thinker. It shows a very rationale thinker trying to cope with the immensity of the social and economic crisis that faced Germany after the treatise of Versailles. So, while his conservative, monarchic, point of view and his anti-semitism are certainly

objectionable that is no ground to make Frege into a lunatic or a paranoid. Similar considerations apply to the way Cantor is depicted but I will not delve into that.

Another instance concerns the historical entry on Hilbert:

Though in outward appearance and behavior Hilbert gave the impression of a paragon of normality and mental health, the way he treated his only son, Franz, raises questions. When the boy was diagnosed with schizophrenia, at age 15, his father sent him off to an asylum, where he spent the rest of his life. Hilbert never visited his son. He died in 1943.(p. 327)

But the historical record shows that Franz was interned at age 21 and was released in 1917 (at which point he would have been 24). Moreover, he lived with his family again after his release (see Reid 1970)

Finally consider what the *Notebook* reports on von Neumann:

There is no evidence that [Russell] was in the audience during Gödel's 'incompleteness' talk—he probably wasn't and Hilbert certainly wasn't, though von Neumann certainly was and did say "it's all over" right after. (pp. 341-342)

The graphic novel has Gödel's presentation take place in Vienna. It is indeed correct that the first presentation of the result took place in Vienna. But John von Neumann was not there. We have the minutes of that meeting in the hand of Rose Rand and there we have in the audience a local Viennese teacher called Robert Neumann (see Stadler 1997 and Mancosu 1999). I wonder if this is the source of the confusion. Before this presentation in Vienna, Gödel stated the first incompleteness theorem in Königsberg in September 1930 in an off-hand remark (but not a presentation) during the general discussion on the last day. Immediately after, he was cornered by John von Neumann who asked for more details. But I know of no source that indicates that von Neumann exclaimed "it's all over". In correspondence with Bernays and Gödel he did however indicate that he thought Gödel's theorems meant for him the failure of Hilbert's program. That sounds more reasonable. (See Mancosu 1999 for documents bearing on this exchange).

## 3.2. The portrayal of intellectual ideas.

I will focus on two issues: a) the characterization of the foundations as the search for certainty and b) Wittgenstein's Tractarian philosophy to which *Logicomix* devotes quite a bit of attention.

### 3.2.1 Foundations as a search for certainty?

Over and over again the foundational quest is portrayed by *Logicomix* as a search for certainty. This is described as Russell's main goal (p. 114, p. 256) not only when he is attempting to settle the problem of foundations in 'Principles of

Mathematics' but well after the writing of *Principia Mathematica*. Whatever the case might be for the early Russell up to 1903 –and there is indeed evidence that the search for certainty in mathematics and natural science might well have been Bertrand Russell's initial impulse to engage in foundational research-- to characterize his foundational work as a search for certainty after that period misses Russell's intellectual development. Indeed, already in Russell 1906, in a lecture in 1907 ("The regressive method of discovering the premises of mathematics", published in Russell 1976, pp. 272-283), and then also quite explicitly in *Principia Mathematica* (p. v and p. 59), Russell points out that foundational work in mathematics does not aim at certainty, which is in principle unreachable, but rather at an explanatory task. Indeed, contrary to what the authors have Russell say in the graphic novel on p. 185, for the real Russell '1+1=2' is more certain to us than any of the logical axioms postulated by *Principia*.

[Picture 2]

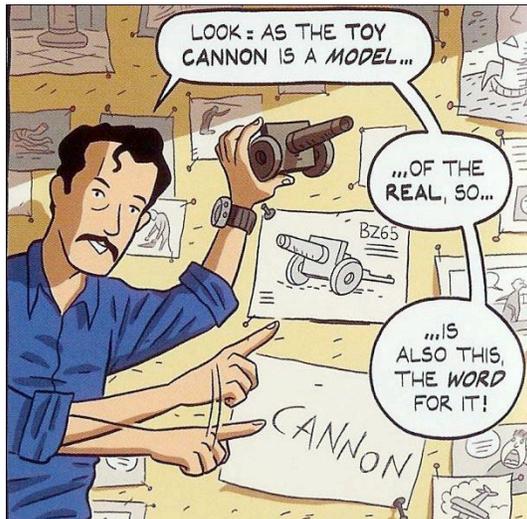


According to him, the role of the axioms of *Principia* is similar to that of the hypotheses in natural science which are not meant to increase the certainty of the empirical phenomena with which we are acquainted but rather to explain them. Incidentally, Gödel held similar positions (see Mancosu 2001 for corroborating evidence that both Russell and Gödel held such views on the role of foundations). I am also at a loss to understand what is meant by claiming, as Alecos does on p. 265 of the graphic novel, that it was the *Tractatus* that shook Russell's dream of certainty.

### 3.2.2 *The Tractatus*.

Concerning the Tractarian theory known as the ‘picture-theory of language’, I should say that I admire the authors for having attempted the challenge of treating it in the story. But there is a point where their presentation misses the mark. The picture theory of language is a theory about how propositions represent or depicts reality. They represent reality because they have in common with it the ‘logical form’. But Wittgenstein is very careful to distinguish the role of *representation or depiction* played by the proposition in representing reality and the role played by the elements that are connected in the proposition (simple names according to Wittgenstein). The role of the simple names is not that of representing or picturing but simply that of standing for or going proxy for the simple objects corresponding to the names. Thus when on p. 256 we are told that as the toy cannon is a model of the real cannon, so also the word ‘cannon’ is a model of the real cannon, the explanation misses the important distinction made by Wittgenstein.

[Picture 3].



Wittgenstein says:

The name is not a picture of the thing named. The proposition *only says something in so far as it is a picture!* (Der Name ist kein Bild des Benannten! Der Satz sagt nur insoweit etwas aus, als er ein Bild ist!) Notebooks, October 3, 1914 (Wittgenstein 1984)

In the *Tractatus* (see 2.131) the same distinction is captured by the opposition between the role of the elements of the picture, *vertreten* (to go proxy for; to stand for), and the role of the picture, *darstellen, vorstellen* (to represent).

Another claim attributed to Wittgenstein, this time in the entry on the *Tractatus* in the *Notebook*, is that he saw “mathematics and logic as machines for producing tautologies” (p. 338) While one can claim that for Wittgenstein logic is tautological, he never made this claim for mathematics.

### 3.3 Some logical inaccuracies

Obviously, *Logicomix* would have missed its intended aim, had it turned into some sort of “logic for dummies” or “logic for poets”. The intent of the authors was not to teach technical material to the reader. However, since technical results play a large part in the intellectual quest, they rightly added in the *Notebook* sections describing the nature and import of such results. It is unfortunate that in some cases the technical details are not correct. Hopefully, this can be changed in a second edition of the graphic novel. I will only mention the two most important occurrences. The first occurs in the description of Gödel’s incompleteness theorems on p. 328. The text reads:

**Incompleteness Theorem.** In 1931, the 25 years old Kurt Gödel proved two theorems that are sometimes referred to as “the” Incompleteness Theorem –though occasionally this form is used to denote the first of these. The completeness of a logical system is the property that every well-formed (i.e. grammatically correct by the rules of the system) proposition in it can be proved or disproved from the system’s axioms. Gödel’s earlier *Completeness Theorem* shows that there is a simple such axiomatic system for first order logic.”(p. 328)

But this description confuses two different notions of completeness, syntactic (the one relevant to Gödel’s incompleteness theorems) and semantic (the one relevant to the semantical completeness of first-order logic). First-order logic is semantically complete but not syntactically complete (for instance there are exactly two objects is neither provable nor disprovable from first-order logic).

The second case I want to mention, I would like to characterize as misleading in the extreme. In the entry on “Predicate calculus” we read:

Rigorously defined, the version of predicate calculus called *first-order logic* employs simple mathematical objects as variables, whereas in second-order logic variables can also be sets, making possible statements like “there is a set S”. This, more powerful language, can express all known mathematics. (p. 333)

In first order logic variables range over objects in a given domain; in second-order logic we also have variables that range over the subsets of that given domain. But this is not captured by the above characterization which, if taken at face value, seems to say that in first-order logic we cannot have variables ranging over sets. But obviously we do that all the time in theories such as first-order ZFC (Zermelo-Fraenkel with Choice). Incidentally, it is first-order ZFC that is considered powerful enough to express most of classical mathematics. The claim that one needs to go to second-order logic for expressing “all known”

mathematics is undermined in this context. If we need to go to second-order then we will need to go to even higher-orders.

### 3.4 Madness and Tragedy

I will conclude by expressing my discomfort, respectively puzzlement, with two topics that run through the book: 1) logic and madness; 2) tragedy. I will do so by raising two issues:

a) So why should logic be more closely connected to madness than any other areas of human activity? The rate of mental illness in the general populations is rated at 6%. Consider now Mancosu, Zach, Badesa (2009), a comprehensive article on the history of mathematical logic from 1900 to 1935. Of the some 66 people working in logic/the foundations of mathematics in the period 1900-1935 mentioned there, we know of only four cases of those with periods of mental illness: Cantor, Schönfinkel, Post and Gödel. In other words, a rate of approximately 6%, which does not seem exceptional. I will grant that I have not studied the medical history of all the these people, nor is it clear what the reference class should be, but my question still stand. Do we have any hard evidence on this issue? Or is this simply mythology? My discomfort (2) has its source in the danger that my area of investigation, logic and the foundations of mathematics, should be gratuitously associated with mental illness and paranoia. This is likely to have negative reverberations with at least some of the reader of *Logicomix*. In light of the above, I find the suggestion running through *Logicomix* that there is an intrinsic association between madness and the pursuit of certainty in the foundations of mathematics to be unfounded, misleading, and potentially damaging.

b) *Logicomix* states (see especially p. 305) but does not make explicit the parallel between the *Oresteia* and the quest for foundations. The explanation given on p. 305 relates the *Oresteia* to historical events in Europe but not to the debate on foundations. So, all in all, I do not find that the authors make the parallel in a convincing way.

### Conclusion.

I enjoyed reading *Logicomix* immensely. The authors have tackled an extremely complicated subject with thought provoking ideas in an aesthetically pleasing and entertaining fashion. Thus, my few critical remarks should not mislead you. I highly recommend *Logicomix* even though my recommendation is qualified: the reader should provide his/her grain of salt.

**Acknowledgements.** I would like to thank Richard Zach for conversations on the contents of his review and Solomon Feferman for his comments on a previous draft.

## Notes.

1. The standard biography of Russell is the two volume work by Ray Monk (Monk 1996, 2000). See also Russell 1967.
2. Shared with others such as Richard Zach in <http://www.ucalgary.ca/~rzach/logblog/2009/09/logicomix-epic-search-for-truth.html>

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