FOL translation exercises

1. Look carefully at how the following sentences have been translated into FOL. (You should try to do them on your own first.)

(a) **Each cube is to the left of a tetrahedron.**

   Actually this is ambiguous. Here is the weak version: \( \forall x (\text{Cube}(x) \rightarrow \exists y (\text{Tet}(y) \land \text{LeftOf}(x,y))) \)

   Here is the strong version: \( \exists x (\text{Tet}(x) \land \forall y (\text{Cube}(y) \rightarrow \text{LeftOf}(y,x))) \)

(b) **No cube to the right of a tetrahedron is to the left of a larger dodecahedron.**

   This is a terrible sentence. It is doubly ambiguous. Is there just one tetrahedron, or could there be many? (I say the latter in my translation.) Is the dodecahedron larger than the tetrahedron, or than the cube (I again say the latter). Don’t mix up the variables in what follows! Exercise: give translations reflecting the other possible meanings of the English.

   \( \forall x ((\text{Cube}(x) \land \exists y (\text{Tet}(y) \land \text{RightOf}(x,y))) \rightarrow \neg \exists y (\text{Dodec}(y) \land \text{Larger}(y,x) \land \text{RightOf}(x,y))) \)

(c) **If a freshman takes a logic class, he or she is smart.**

   \( \forall x ((\text{Freshman}(x) \land \exists y (\text{LogicClass}(y) \land \text{Takes}(x,y))) \rightarrow \text{Smart}(x)) \)

(d) **Every farmer who owns a donkey beats it.**

   \( \forall x (\text{Donkey}(x) \rightarrow \forall y ((\text{Farmer}(y) \land \text{Owns}(y,x)) \rightarrow \text{Beats}(y,x))) \)

   The following is also okay: \( \forall x \forall y ((\text{Farmer}(x) \land \text{Donkey}(y) \land \text{Owns}(x,y)) \rightarrow \text{Beats}(x,y)) \)

2. **Ex 11.18** This was homework so I won’t give answers here, but I will answer questions you have about it during office hours.

3. **Ex 11.19**

   (a) **Every dodecahedron is as large as every cube.**

   \( \forall x (\text{Dodec}(x) \rightarrow \forall y (\text{Cube}(y) \rightarrow \neg \text{Smaller}(x,y))) \)
(b) If a cube is to the right of a dodecahedron but not in back of it, then it is as large as the dodecahedron.
$$\forall x \forall y ((\text{Cube}(x) \land \text{Dodec}(y) \land \text{RightOf}(x,y) \land \neg \text{BackOf}(x,y)) \rightarrow \neg \text{Smaller}(x,y))$$

(c) No cube with nothing to its left is between two cubes.
$$\forall x (\text{Cube}(x) \rightarrow (\neg \exists y \text{LeftOf}(y,x) \rightarrow \neg \exists z \exists w (\text{Cube}(z) \land \text{Cube}(w) \land \text{Between}(x,z,w))))$$

(d) The only large cubes are $b$ and $c$.
Note: unlike the next sentence, this one implies that $b$ and $c$ are large cubes.
$$\text{Cube}(b) \land \text{Large}(b) \land \text{Cube}(c) \land \text{Large}(c) \land \forall x ((\text{Cube}(x) \land \text{Large}(x)) \rightarrow x = b \lor x = c)$$

(e) At most $b$ and $c$ are large cubes.
$$\forall x ((\text{Cube}(x) \land \text{Large}(x)) \rightarrow x = b \lor x = c)$$

Alternately: $$\neg \exists x (\text{Cube}(x) \land \text{Large}(x) \land x \neq b \land x \neq c)$$