PHIL12A
Section answers, 30 March 2011
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1 How much do you know?

1. Translate the following sentences (these are the four ‘Aristotelian forms’). For each sentence, come up with a translation that uses the ∀ quantifier and one that uses the ∃ quantifier:

(a) All cubes are small.
   \[ \forall x (\text{Cube}(x) \rightarrow \text{Small}(x)) \]
   \[ \neg \exists x (\text{Cube}(x) \land \neg \text{Small}(x)) \]

(b) Some cubes are small.
   \[ \exists x (\text{Cube}(x) \land \text{Small}(x)) \]
   \[ \neg \forall x (\text{Cube}(x) \rightarrow \neg \text{Small}(x)) \]

(c) No cubes are small.
   \[ \neg \exists x (\text{Cube}(x) \land \text{Small}(x)) \]
   \[ \forall x (\text{Cube}(x) \rightarrow \neg \text{Small}(x)) \]

(d) Some cubes are not small.
   \[ \exists x (\text{Cube}(x) \land \neg \text{Small}(x)) \]
   \[ \neg \forall x (\text{Cube}(x) \rightarrow \text{Small}(x)) \]

2. Given any sentence starting with ∀, how can you arrive at an equivalent sentence in which the ∀ symbol has been replaced with the ∃ symbol? And if you have a sentence starting with ∃, how could you arrive at an equivalent sentence in which the ∃ symbol has been replaced with the ∀ symbol?

Let \( P(x) \) be some well formed formula with \( x \) free. Then:

\[ \forall x P(x) \iff \neg \exists x \neg P(x) \]

and

\[ \exists x P(x) \iff \neg \forall x \neg P(x) \]
2 Slightly harder...

1. Translation with multiple quantifiers:

(a) There are some dodecahedra that are small, and some that are large, but none are medium.
\[ \exists x \text{Dodec}(x) \land \forall x (\text{Dodec}(x) \rightarrow ((\text{Small}(x) \lor \text{Large}(x)) \land \neg \text{Medium}(x))) \]

(b) Every large cube has a small tetrahedron on its left.
\[ \forall x ((\text{Cube}(x) \land \text{Large}(x)) \rightarrow \exists y (\text{Tet}(y) \land \text{Small}(y) \land \text{LeftOf}(y, x)) \]

(c) If there is a large tetrahedron, then there are no cubes behind it.
\[ \exists x (\text{Tet}(x) \land \text{Large}(x)) \rightarrow \forall x ((\text{Tet}(x) \land \text{Large}(x)) \rightarrow \neg \exists y (\text{Cube}(y) \land \text{BackOf}(y, x)) \]

(d) There is more than one large cube.
\[ \exists x \exists y (\text{Large}(x) \land \text{Cube}(x) \land \text{Large}(y) \land \text{Cube}(y) \land x \neq y) \]

(e) There is exactly one large cube.
\[ \exists x \forall y (\text{Large}(x) \land \text{Cube}(x) \land ((\text{Large}(y) \land \text{Cube}(y)) \rightarrow y = x)) \]

(f) There are exactly two large cubes.
\[ \exists x \exists y \forall z (\text{Large}(x) \land \text{Cube}(x) \land \text{Large}(y) \land \text{Cube}(y) \land x \neq y \land ((\text{Large}(z) \land \text{Cube}(z)) \rightarrow (z = x \lor z = y))) \]