1 How much do you know?

1. (Ex 2.22) Is the following argument valid? Sound? If it is valid, give an informal proof of it. If it is not, give an informal counterexample to it.

   All computer scientists are rich. Anyone who knows how to program a computer is a computer scientist. Bill Gates is rich. Therefore, Bill Gates knows how to program a computer.

   This is invalid, and therefore we know immediately that it cannot be sound. (Why?) Here is a counterexample showing that it is invalid: suppose the world contains exactly four people. All of them are rich. Exactly three of these people know how to program a computer, and they are all computer scientists. The other person is Bill Gates, who does not know how to program a computer and is not a computer scientist. Now the following sentences are true:

   - All computer scientists are rich.
   - Anyone who knows how to program a computer is a computer scientist.
   - Bill Gates is rich.

   However, it is not true that Bill Gates knows how to program a computer. In other words, we have shown that there is a possible world in which all the premises are true and the conclusion is false.

   If this was not obvious to you, take a close look at the first premise, which has the form ‘All As are Bs.’ This means that if someone is an A, then they are a B. It does not mean that if someone is a B, they are also an A. So all computer scientists are rich does not mean that all rich people are computer scientists.

2. Write formal proofs for the following, as you would using the program Fitch. You can use the following rules: =Intro, =Elim, Reit, Ana Con. Make sure you cite sentences to justify your use of the rule in each case.
(a) (Ex 2.24)

1  Larger(b,c)
2  Smaller(b,d)
3  SameSize(d,e)
4  Larger(e,c)

We need to use Ana Con here a bunch of times, since there are no premises using the identity relation. Make sure you understand why each inference using Ana Con holds, and that you can explain in words why it holds. Make especially sure you can see why I have cited the lines I have for each use of Ana Con. Here is a possible proof:

1  Larger(b,c)
2  Smaller(b,d)
3  SameSize(d,e)
4  Larger(d,b)  Ana Con: 2
5  Larger(d,c)  Ana Con: 4,1
6  Larger(e,c)  Ana Con: 5,3

(b) (Ex 2.27)

1  SameRow(b,c)
2  SameRow(a,d)
3  SameRow(d,f)
4  FrontOf(a,b)
5  FrontOf(f,c)

Here is a possible proof. Again, make sure you understand why I have cited the lines I have for each use of Ana Con.
2 Something slightly harder, if there’s time.

1. Consider the sentence $\phi$, of unknown truth value. $\psi$ is a sentence that has been formed by adding some number of negation symbols in front of it. $\psi$ has the same truth value as either $\phi$ or $\neg \phi$. Describe a mechanical procedure by which you can determine this truth value.

Here’s an algorithm you can teach your computer that will take the string of symbols making up $\psi$ and either output $\phi$ or $\neg \phi$, whichever is equivalent to $\psi$, or will output an error message:

(a) Read the first symbol. If it is $\phi$ then output $\phi$ and stop. If the first symbol is $\neg$, then set the counter to 1 and proceed to the next step. If the first symbol is neither $\phi$ nor $\neg$, then output an error message and stop.

(b) Read the next symbol. If it is $\neg$ then proceed to step (c). If it is $\phi$ then proceed to step (d). If it is neither $\neg$ nor $\phi$ then output an error message and stop.

(c) If the counter is set to 1, then set the counter to 0 and repeat step (b). If the counter is set to 0, then set the counter to 1 and return to step (b).

(d) If the counter is set to 1, then output $\neg \phi$ and stop. If the counter is set to 0, then output $\phi$ and stop.

You didn’t need to answer the question like this. If you can explain informally how to do it that is fine.

The basic insight is that pairs of $\neg$ symbols cancel each other out. So count up the number of such symbols: if there an even number, then $\psi$ is equivalent to $\phi$, otherwise it is equivalent to $\neg \phi$.

So, what the algorithm does is to flip-flop between recording a 0 or recording a 1 as it reads the symbols that make up $\psi$. When it finally hits $\phi$ it checks what the counter is set to. If it read an even number of $\neg$ symbols, then the counter will be set to 0, and the algorithm outputs $\phi$. Otherwise it read an odd number of $\neg$ symbols, and so it outputs $\neg \phi$. 
2. Are the following arguments valid or invalid? Explain why. Are they sound?

(a)

1. Julian loves logic.
2. Julian does not love logic.
3. Julian speaks at least six languages.

Actually, I do not speak at least six languages. But you didn’t need to know this. Either way the argument is valid!!!

Here’s why. Remember the definition of validity: an argument is valid if and only if it is impossible to make all the premises true and the conclusion false. Now, in this case the premises contradict it each other. If premise 1 is true then premise 2 is false. If premise 1 is false, then premise 2 is true. In other words, it is impossible to make the premises true. But then it is also impossible to make the premises true and the conclusion false.

Work through this until you understand it. When you do, you will understand the definition of logical validity. You don’t have to agree with the definition in order to understand it!

Since the argument is valid, it would be sound if the premises were true. But we cannot make both the premises true. So it is not sound, whether I love logic or not.

(b)

1. 2+2=5
2. 2+2=4

Believe it or not, this one is also valid. Recall the definition of validity: the argument is valid if and only if it is impossible for the premise to be true and the conclusion false. In this case it is impossible for the premise to be true, which means that it is impossible for the premise to be true and the conclusion false.

In fact, we could have argued differently. It is impossible for the conclusion to be false, since 2+2=4 in every possible world. So it is impossible for the premise to be true and the conclusion false.

Is the argument sound? It is valid, so it would be sound if the premise were true. But this premise could never be true, so the argument is not sound.

(c)

1. 2+2=4
2. 3+3=6
3. The moon is made of green cheese.

This argument is invalid. The premises are always true, and there is a possible world in which the conclusion is false (perhaps it is false in this world). Since the argument is invalid, it cannot be sound.
3 Challenge question.

Consider the following English sentences, which are composed of two or more simple sentences connected by the boldface sentential connectives. In each case, decide whether the connective is truth-functional or not. (A connective is truth-functional if the truth value of the whole sentence depends on the truth value of the simple sentences it joins and nothing else.)

In each case, if the connective is truth-functional, draw up a truth table for it. If the connective is not truth-functional, come up with (an) example sentence(s) that shows that it is not truth-functional. (What exactly would you have to show?)

You have seen some truth tables by now in the textbook. Make sure that you list all possible combinations of truth values. If your sentential connective takes two sentences, then each sentence can take on one of two truth values, so the total number of lines in the truth table should be $2 \times 2 = 4$. If the connective takes three sentences, its truth table would have $2 \times 2 \times 2 = 8$ lines.

How do you show that a connective is not truth-functional? You need to show that the value of the entire sentence does not depend solely on the truth of the simple sentences it is made of. The following would do: two possible worlds, one of which makes the premises true and the conclusion true, and the other of which makes the premises true and the conclusion false.

1. Obama is President just in case most voters voted for Obama.

This is a truth-functional connective, since the truth value of the whole sentence depends only upon the truth values of the sentence on the left of the connective and the sentence on the right of the connective. Here is the truth table:

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p$ just in case $q$</th>
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<tbody>
<tr>
<td>True</td>
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</table>

2. Elvis got fat before Obama was elected President.

Elvis got fat, and Obama was elected President, but Elvis got fat decades before Obama was voted in. The sentence to the left of ‘before’ is true, and the sentence to the right of ‘before’ is true, and the entire sentence is true.

This is not a truth-functional connective, since its truth does not depend only on the truth values of the simple sentences ‘Elvis got fat’ and ‘Obama was elected President’, but also on the temporal ordering of the things these sentences describe. We can show this by coming up with two sentences, both true, that when connected by ‘before’ make the entire sentence false.

Consider the sentence ‘Obama was elected President before Elvis got fat’. Here, we again have two sentences...
that are true, since indeed Obama was elected President, and indeed Elvis got fat. But it is not true that Obama was elected President before Elvis got fat.

3. The glass broke because I dropped it.

This is *not* a truth-functional connective, since its truth does not depend only on the truth values of the simple sentences ‘The glass broke’ and ‘I dropped the glass’, but also on whether my dropping the glass actually caused the glass to break. Consider the following two possible scenarios:

- I dropped the glass, and on hitting the floor it broke into a zillion pieces. Clearly my dropping the glass caused it to break. ‘The glass broke’ is true, ‘I dropped the glass’ is true, and it is true that ‘The glass broke because I dropped it.’

- I dropped the glass, but was relieved to see that it didn’t break. I picked it up and put it back on the counter. At that moment my would-be assassin bullet fired a bullet through the window, narrowly missing me but shattering the glass. ‘The glass broke’ is true, ‘I dropped the glass’ is true, but it is not true that ‘The glass broke because I dropped it.’

4. At least one of the following three things is true: Elvis is not dead, the moon landings never happened, or the earth is flat. (This is a connective taking three sentences rather than two.)

This is truth-functional, since it depends only on the fact that at least one of the three sentences it conjoins is true. Here is the truth table:

<table>
<thead>
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<th></th>
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<th>At least of the following three things is true: p, q or r</th>
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<tbody>
<tr>
<td>p</td>
<td>q</td>
<td>r</td>
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Use the table to determine what value the example sentence has in the actual world.

5. Necessarily 2+2=4. (Could this be a one place connective, like negation?)

This is not truth functional, but we can’t show this in the way we have done for the previous examples. The entire sentence takes the form ‘Necessarily p’. We must show that for some true p the whole sentence is true, and for some true p the entire sentence is false: in other words, the truth value of the whole does not depend on the truth value of its one part.
It is true that 2+2=4, and indeed it is necessarily the case that 2+2=4, since this is true in every possible world. But consider the sentence ‘Necessarily Obama is President’. It is true that Obama is President, but this is not a necessary truth, since there is a possible world in which someone else (say, Sarah Palin) is currently President.