Advice-Giving and Scoring-Rule-Based Arguments for Probabilism

Lara Buchak & Branden Fitelson

Department of Philosophy
University of California–Berkeley

buchak@berkeley.edu
branden@fitelson.org

By “Scoring Rule Arguments” for probabilism, I mean:
- B is being in an “epistemically dominated” doxastic state (I’m being vague here, but I will return to this B in the end).
- F is that b is inadmissible, i.e., ∃ a set of propositions P (which S entertains) and a (coherent) d.o.b. function b* on P such that b* dominates b on P (wrt a “good” scoring rule s).

Since SR theorems deliver specific (families of) dominating alternative d.o.b. functions b* (on P), one might be tempted to read them as yielding reasons to adopt (some) b* (on P).

That’s a more specific sort of reason than a mere reason to adopt some coherent b’ (on P), as in traditional arguments.

Analogy (to which I’ll return): having reason to adopt some β’ that is logically consistent on P vs having reason to adopt some β* in a specific family of β’s that are consistent on P.

The rest of my talk is a cautionary tale about putting scoring rule arguments to this more specific sort of use.

The tale involves 3 theorems, an example, and an analogy.

Arguments for probabilism can be thought of as ways of delivering (various sorts of) “advice” for incoherent agents.

Standard arguments for probabilism are all of the form:
- An agent S has a non-probabilistic degree of belief function b iff (⇒) S has some “bad” property B (presumably, in virtue of the fact that their b has a “bad” formal property F).

These arguments rest on Theorems (⇒) and Converse Theorems (⇐): b is non-Pr ⇐ b has “bad” formal prop. F.

Dutch Book Arguments. B is susceptibility to sure monetary loss (in a certain betting set-up), and F is the formal role played by “bad” p’s in the DBT and the Converse DBT.

Representation Theorem Arguments. B is having preferences that violate some of Savage’s axioms (and/or being unrepresentable as an expected utility maximizer), and F is the formal role played by “bad” p’s in the RT.

To the extent that we have reasons to avoid these B’s, these arguments provide reasons (not) to have a(n) (in)coherent b.

Scoring Rule arguments seem to yield more specific advice.
A (Dis)Analogy & A Proposal

Two Theorems

An Example

A (Dis)Analogy & A Proposal

Preliminaries

One might use a scoring rule theorem (via, say, Theorem 2) to try to generate specific advice for an incoherent agent $A$.

Suppose $S$ (with language $L$) has a non-probabilistic $b$.

Then, by Theorem 2, there will be a partition $P$ of $L$ on which $b$ is incoherent (i.e., on which $b$ does not sum to one).

So, by your favorite partition-level scoring rule theorem, there will exist some $b^*$, which (a) is coherent on $P$, and (b) dominates $b$ on $P$ (under some "good" scoring rule $s$). And, (conversely) no such $b^*$ will be dominated in this way.

At this point, you might be tempted to conclude that $S$ has reason to adopt (some) $b^*$ (on $P$).

The following example suggests that this may be hasty.

Consider an agent $S$ with a 2-atomic-sentence $(X,Y)$, and a d.o.b. function $b$ on $L$, which satisfies these six constraints:

\[
\begin{align*}
 b(X \land Y) &= \frac{1}{10} \\
 b(X \land \neg Y) &= \frac{2}{5} \\
 b(X \lor Y) &= \frac{3}{10} \\
 b(X) &= \frac{1}{2} \\
 b(Y) &= \frac{3}{5} \\
 b(Y) &= \frac{2}{5}
\end{align*}
\]

Note that $b$ is coherent on the partition of state descriptions of $L$, but $b$ is incoherent on two other partitions of $L$.

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Consider an agent $S$ (with $\beta_A$) in a (global) preface case.

Does $S$ have reason to adopt some $\beta_A^*$ that’s consistent on $A$?

Does $S$ have reason to adopt some $\beta_A^*$ from a specific family of $\beta$’s that are logically consistent on $A$?

Perhaps $S$ has some reason to adopt some consistent $\beta_A^*$, since that’s the only way for $S$ to avoid being such that she knows a priori that some of her beliefs are false ("bad" $B$).

But, it doesn’t seem that $S$ need have any reason to adopt a $\beta_A^*$ from any specific family of consistent $\beta$’s. [Suppose $S$ has no evidence indicating any particular (sets of) beliefs are false.]

Now, return to an agent $S$ with a $b_A$ that is incoherent on $A$.

Does $S$ have reason to adopt some $b_A^*$ that’s coherent on $A$?

Does $S$ have reason to adopt a $b_A^*$ from a specific family of $b$’s that are coherent on $A$ (viz, those which $s$-dominate $b_A$ on $A$)?

Disanalogously, scoring rule arguments (if correct) may be able to justify affirmative answers to both questions.

Proposal: talk only about “incoherence on $A$”, and use SRAs — on $A$ — to provide “specific advice” for incoherent agents.

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