# Evidence

## Discrimination, Indication, and Leverage

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# What confirmation theorists want

- To define a relation R(e,h) that holds when evidence e supports hypothesis h (*incremental support)*

- R(e,h) should be a generalization of the relation that obtains when e deductively implies h. I.e., deductive implication should be the strongest kind (limit case) of evidential support.

- Largely settled on the Bayesian approach:
  - Only probability used in the analysis, no other concepts
  - All statements assigned probabilities, including hypotheses, so P(h/e) has a value.

# What more I want in this project (*Tracking Truth*, OUP, 2006)

- R(e,h) should be a *relevance measure*:
  **R(e,h) iff P(h/e) > P(h)**

- R(e,h) should be the best measure of *degree* of support (discrimination, leverage, etc.)

- R(e,h) should fit into a scheme that insures absolute confirmation—that is, P(h/e) is high (indication)—in addition to incremental support

- R(e,h) should respect idea that evidence is leverage.

# Relevance Measures - we have to choose

e supports h iff (equivalently)

P(h/e)/P(h) > 1 (or: P(e/h)/P(e) > 1) *ratio measure*

P(h/e) - P(h) > 0 *difference measure*

**P(e/h)/P(e/-h) > 1** ***likelihood ratio measure***

[P(h/e) - P(h)]/P(e) > 0 *normalized difference measure*

These become measures of support when we say the **higher** the value the **more** the support.

*Measures not ordinally equivalent* – different answers to questions:

Does $e_1$ or $e_2$ confirm h more?

Does e confirm $h_1$ more than it confirms $h_2$?

# Likelihood Ratio: Best for your Money

A. Only measure on the list that
   i. generalizes deduction
   ii. solves quantitative old-evidence problem
   iii. removes the old-hypothesis problem
   iv. judges degree of support by more than fit of h to e
   v. allows account of independence of evidence - 4 properties (Fitelson 2001)

B. Real people use it successfully (Osherson, et al.)

C. Pure discrimination measure

D. Maximizes leverage

E. Makes possible explanatory relation to tracking theory of *knowledge*

# Pure Discrimination

*Why LR measure is better than Ratio Measure:*

| | |
|---|---|
| P(e/h)/**P(e/-h)** | LR Measure |
| P(e/h)/[P(e/h)P(h) + **P(e/-h)**P(-h)] | Ratio Measure |

P(e/-h) occurs in the denominator of the ratio measure, but is diluted by other things, including priors of hypothesis.

→ Ratio measure not a pure measure of e's discrimination of h from –h.

# Leverage

P(e/h)/**P(e/-h)** LR Measure

P(e/h)/[P(e/h)P(h) + **P(e/-h)**P(-h)] Ratio Measure

The dilution of P(e/-h) in the denominator of the ratio measure also prevents maximum leverage.

Occurrence of P(h), P(-h) in denominator means, on usual way of evaluating, that we have to know something about truth of h in order to evaluate how far e is evidence for h.

Note: *Of course* we need priors on hypotheses in order to evaluate P(e/-h). But those hypotheses are *not* h and –h. They are P($h_1$/-h), etc.

# Need Condition insuring High Posterior (High P(h/e))

Options:

1. Require high P(h/e)

   --just restates the desideratum

2. Require high P(h)

   --less leverage than we can get

   --wouldn't be a way in which e *makes* h highly probable.

3. Require high P(e) ======→

# The Proposed Scheme

- Take P(e) as an independent variable, P(h), P(-h) as dependent variables, in Bayes' Equation. Nothing mathematical forbids doing this.
- Require for good evidence high P(e) in addition to high LR

Claims:

1. This gives us high posterior, and that it is *made* high by the evidence.

2. This gives us picture in which leverage property of evidence is maximized.

# The Leverage Equation

P(e), P(e/h), P(e/-h) alone determine posterior, P(h/e):

$$P(h/e) = [LR - P(e/h)/P(e)] \, \mathbf{/} \, (LR-1)$$

Note: P(h/e) maximized when P(e/h) = P(e) because we're moving backwards from the usual scheme.

# Gloss

$$P(h/e) = [LR - P(e/h)/P(e)] / (LR-1)$$

P(h), P(-h) are weights, measuring how well h and –h have predicted the true probability of e:

$$P(e) = P(e/h)P(h) + P(e/-h)P(-h)$$

When P(e/h)/P(e) = 1, h has predicted the true probability of e perfectly. This is why P(h/e) is maximal in that situation, and P(h) is also 1.

Recall: in this scheme we take P(e) as an independent variable, P(h), P(-h) as dependent variables.

Note: it has been assumed we are in the special case in which the likelihoods P(e/h) and P(e/-h) remain fixed. See later example to see why this is a common case.

# Leverage

But, wait:

$$P(e) = P(e/h)P(h) + P(e/-h)P(-h)$$

So, how can this picture have evidence satisfying the leverage requirement? Information about P(h) is in there in information about P(e).

Not so fast. We can know 4 = x + y without knowing x, y.

Leverage is about relationships among *evaluations* of the terms. The question is whether we can *evaluate* P(e) directly, observationally, instead of calculating P(e) out of its components, P(h), P(-h), likelihoods.

Equation alone doesn't say which direction the evaluation must go.

# Application

**D**, an infectious disease
**N**, a newly found, remote population strongly suspected of exposure to **D**
We have a limited amount of drug for treatment.
If **D** is advanced enough for symptoms to show, treatment doesn't work→ death

**h** = x has disease **D**
**e** = x tests positive for disease **D**
We want to find P(**h**/**e**) for x = Mary.

(Thanks to Eric Christian Barnes for this example of how the current scheme can be both sound and distinctively useful.)

We don't know P(**h**), P(**-h**), and can't, because we have no statistics on the prevalence of disease in this population, and waiting for this information will kill the people who are infected.

We have other populations to treat, so we don't want to waste the drug.

→ We can't apply Bayes Theorem forward: P(h/e) = P(e/h)P(h)/P(e), because we don't have priors P(h) and P(-h).

→ We have a problem.

# Application

We can't apply Bayes Theorem forward: P(h/e) = P(e/h)P(h)/P(e), because we don't have priors P(h) and P(-h).

We do have fixed likelihoods, though, because we have a blood test with known sensitivity and specificity, which gives us P(e/h), P(e/-h).

P(**e**/**h**) = .95

P(**e**/**-h**) = .15

→ **LR** = 6.33

**h** = x has disease **D**

**e** = x tests positive for disease **D**

We want to find P(**h**/**e**) for x = Mary.

SInce likelihoods are fixed, we can apply the leverage equation if we can independently find values for P(**e**).

P(**h**) would come from observed frequency of disease in population (which we don't have), so P(**e**) should be observed frequency of *positive testing*.

We give everyone the test, and use P(**e**) and likelihoods to calculate posterior via the Leverage Eqn. for x = anyone who tested positive. For anyone who tested negative we use the leverage equation with **-e**.

# Problems and Questions

How do we do updating on this scheme?

How do we incorporate data about P(h), P(-h) when we *do* get some?

How do likelihoods get updated?

How does P(h/e) we have learned enter into P*(h) of next round?

One thing we don't do: conditionalize on some counterfactual e, e.g. {K-e}